COMPUTATION AND SCIENCE FOR TEACHERS

PITTSBURGH SUPERCOMPUTING CENTER

Water, Water, Everywhere

Introduction to Computational Reasoning

Dependencies in Systems Models

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This module is an introduction to computational reasoning. We are going to create a systems model of a real world experiment, observing how fractions can be used to calculate how the system changes over time. We will use the model to compare a closed system to an open one. No prior knowledge of computer modeling or Excel is needed.

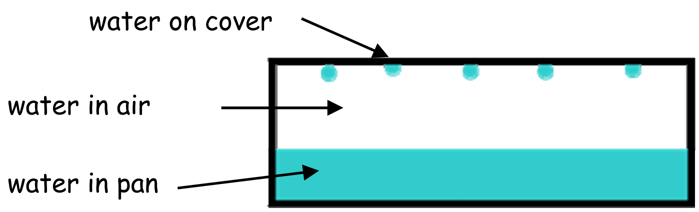
Water, Water, Everywhere

Cycles, equilibrium and conservation of mass are three important concepts in science. Using fractions to represent the proportion of an amount that goes through some kind of change is fundamental to mathematical modeling of systems. In this example, we will use fractions and the water cycle to illustrate equilibrium and conservation of mass.

The Physical Problem

What happens to water in a closed pan put on a stove to boil?

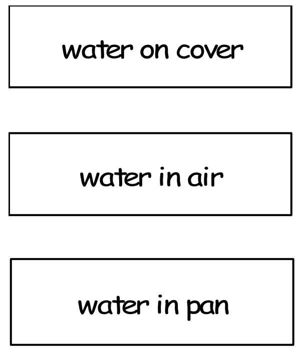
At first, the cover is dry and the air has little moisture. However, as the water heats, some of it evaporates into the air, some of that water vapor condenses on the lid, and some of the water vapor on the lid precipitates to fall back into the water in the pan. If we had a clear container, we could see evidence of those processes. In a classroom, we could illustrate the process by placing water in a foil pie pan with a clear plastic lid which would then be placed over a heat source. As the heat causes some of the water to evaporate, the students see water collect on the cover.



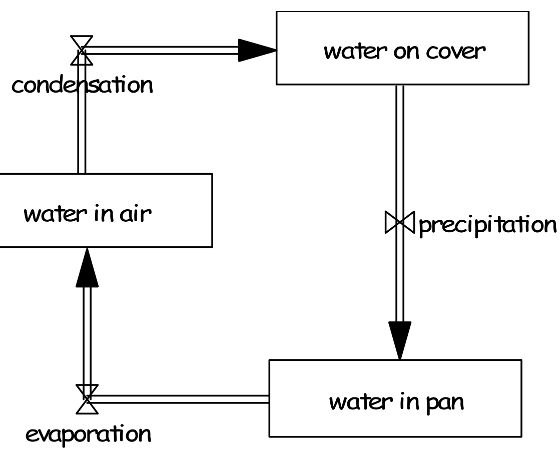
But how much water is in the air and how much is on the cover?

A Systems Model in Vensim

Since it is very difficult to measure those quantities directly, we'll use a model to represent the system. We'll start with 3 box variables to represent the amounts of water in the pan, in the air and on the cover.



Next, we'll connect the 3 boxes with pipes representing the rates at which the water moves from one box to the other. These pipes are called evaporation, condensation, and precipitation. Note that the boxes have been rearranged to make it easier to connect them with pipes.



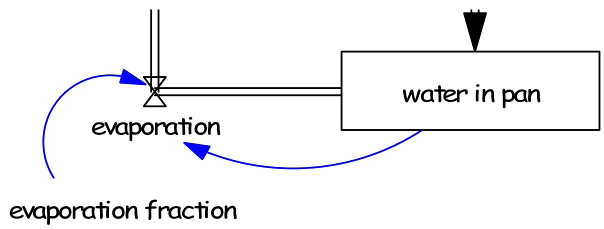
How should precipitation affect the amount of water in the pan?

How should evaporation affect the amount of water in the pan?

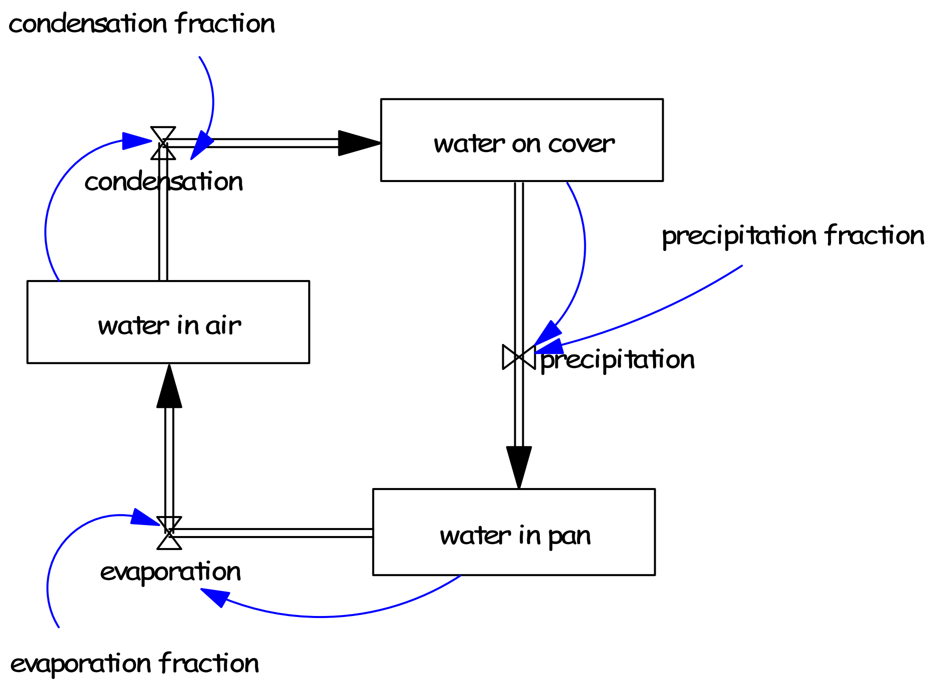
Since the diagram shows the precipitation pipe pointing into the water in pan box and the evaporation pipe pointing out of the water in pan box, we know that precipitation is added to water in pan and evaporation is subtracted from water in pan. Similar addition and subtraction computations occur throughout the cycle.

What isn't visible is that these computations take place over time. We're going to measure time in minutes. We know that as the pan heats up, a fraction of the water in the pan will evaporate into the air each minute. Once the air becomes saturated, a fraction of the water in the air will condense on the cover each minute. Finally, a fraction of the water on the cover will fall back into the pan each minute. We call these fractions: evaporation fraction, condensation fraction, and precipitation fraction.

Let's look just at evaporation. Suppose the evaporation fraction is 1/10. That would mean that 1/10 of the water in the pan evaporates into the air each minute. That means that our model needs to show that we need to know the water in the pan and the evaporation fraction in order to compute the evaporation rate. We do that by adding the evaporation fraction variable and arrows connecting water in pan and evaporation fraction to the evaporation pipe, as seen below.



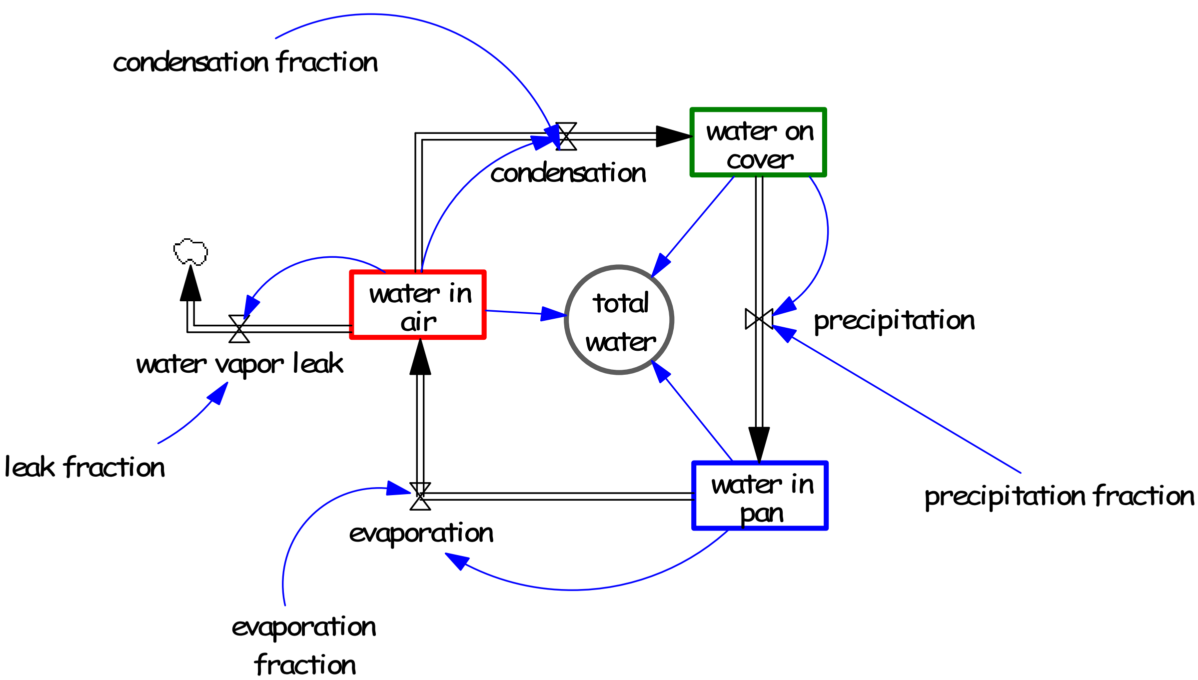
After doing this for all three rates in the model, our diagram looks like this:



Have we left anything out? What assumptions have we made in this model?

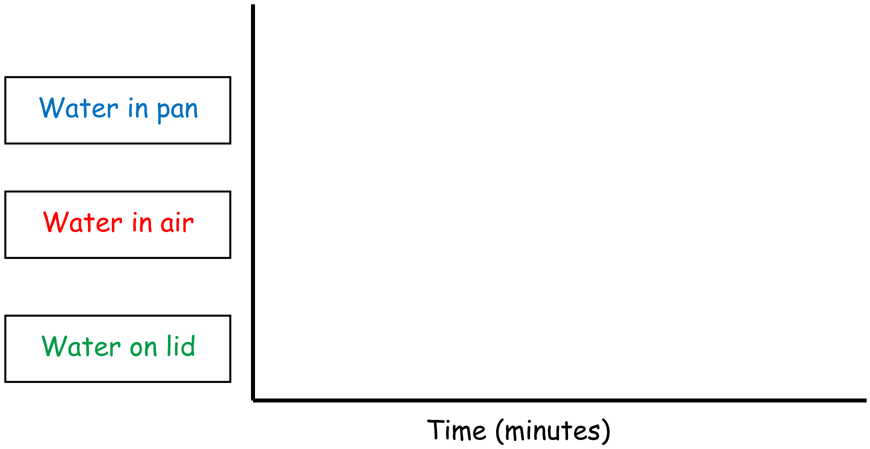
The model as we've drawn it is closed. There are no leaks in any part of the system. If that is the case and we start with 100 mL of water in the pan, how much water should be in the system throughout the time that we observe it?

To find out whether our closed system conserves mass, we'll add a variable called total water which will be the sum of the amounts of water in the three box variables. To test the effect of a leak in our system, we'll add a water vapor leak. The diagram for the model now looks like this:



Before we actually run this model, let's do some predictions. If the system is perfectly closed and we start with 100 mL of water in the system, how much total water will you expect in the system if we subject the pan to moderate heat for 10 minutes?

If we start with 100 mL of water in the pan and no water in the air or on the lid and we track the amounts of water in the pan, in the air and on the lid over a 10 minute time period, what do you expect the graph to look like? Use the axes below to make a rough sketch.



Let's see what the model says. Download pan_water_cycle_CAST.mdl and double-click to open the file which will launch the application Vensim PLE, free simulation software from Ventana Systems (see http://www.vensim.com/venple.html)

If you click on the green running man with a lined background (to the right of the Current box), a graph (PAN-COVER-AIR Content over time) will be drawn and the setting for all of the fractions will be shown. Notice that the leak fraction is 0, the evaporation fraction is 0.1, and the condensation and precipitation fractions are both 0.5.

What does the graph tell you about the behavior of this closed system?

Note that each variable in the diagram is overlaid with a mini-graph showing its behavior over the 10 minute runtime.

Let's introduce a leak into our system. How do you expect the graph to change?

Note that as you increase the leak fraction slider, all of the mini-graphs have two curves on them, the original curve in red and the new curve in blue. The large graph shows only the new curves. Using the sliders on the fractions, you can vary them one at a time to see the effect on the graph. Singly or in groups of four, experiment with the four fractions to discover how the system behaves.

The Model in Excel

Now, let's look at the same model executed in Excel. Open pan_water_cycle_CAST.xls.

Go to the tab labeled Model 1. Here you see an interactive Excelet that is based on the Vensim model we just explored. The precipitation, condensation and evaporation fractions can be controlled by using the sliders. Note that as you use the sliders, the graph changes just as we saw in the Vensim model.

How do we introduce a leak into the system? Go to the Model 2 tab where you will see that a leak fraction slider has been added to the model. Manipulate that slider to see the effect of a leak on the water cycle.

The Math behind the Model

In both the Vensim and Excel versions of the systems model, we have seen the output in the form of a graph. But, those graphs are the result of plotting numbers that represent the number of mL of water in the pan, in the air, and on the cover over a number of minutes. To see how those numbers are calculated, scroll to the right on the Model 2 sheet of the Excel model. There you will see the three equations that are used to model the system.

The first equation (in blue) tells us that the amount of water we have in the pan at a certain time is equal to the amount that we had at the time step earlier plus the amount of precipitation from the lid minus the amount of evaporation from the pan during that time step. The concept that the amount you HAVE is the amount you HAD plus CHANGE is the fundamental concept underlying any systems model. The second and third equations work in exactly the same way.

Acknowledgments

The original idea for this model came from the work done by Mary Ellen Verona and Don Higdon during the MVHS CoreModels Project.