Exploring Radioactive Decay

Use the accompanying interactive Excel spreadsheet to address the questions that follow. The yellow cells and sliders (◄ | ►) are adjustable variables. First let's consider the parent isotope decaying to a stable or non-radioactive daughter product isotope or P → D. Click on the decay tab (lower left of screen) of the multi-worksheet spreadsheet.

1.   How does changing the half-life influence the A/A_o versus time graph? Sketch, label the axes, and label the graphs of two different half-lives.


2.   Which curve above has the greater decay constant (rate constant)?

3.   Does changing the starting activity of the parent isotope influence the A/A_o versus time graph? Explain why or why not.

4.   Suppose that a substance has a very large half-life. Could a graph of A/A_o versus time be mistakenly interpreted as a linear graph? Why or why not?

5.   The slope of the ln A/A_o versus time graph is minus the decay constant, -k. Why is the slope of the log A/A_o versus time graph different?

6.   What is the slope equal to for the log A/A_o versus time graph?

7.   If you were <u>analyzing data using a hand-drawn graph</u> to determine values such as slope or half-life, which type of plot (A/A_o versus time or the logarithm, either log or ln of A/A_o versus time) would you select to most accurately determine the half-life? Explain your selection.

8.   Determine the half-life from the log A/A_o versus time and ln A/A_o versus time graphs given below for the <u>same radioactive parent isotope</u> decaying. Show how you determined it on both graphs.
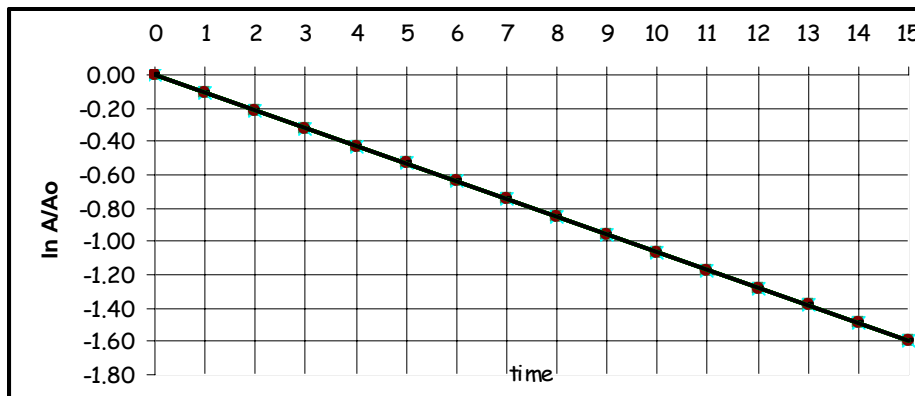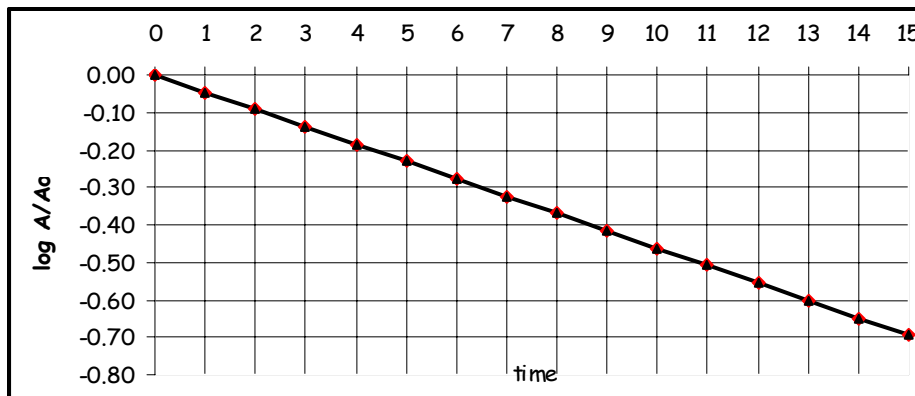
Here is the comparison of two isotopes ($A/A_o = e^{-kt}$) with different half-lives.
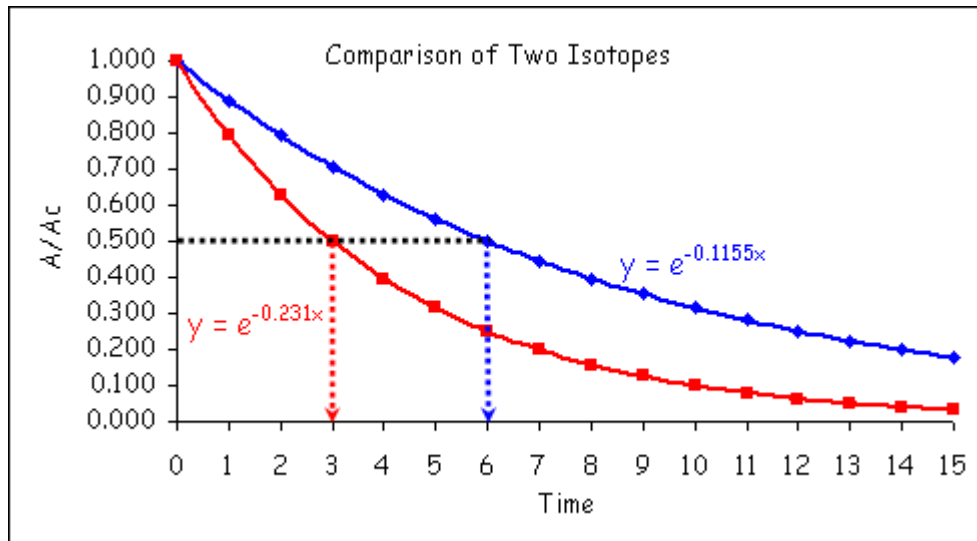
Since half-life ($t_{1/2}$) and the decay constant (k) are related by $t_{1/2} = \dfrac{0.693}{k}$, the longer the half-life, the smaller the decay constant.

The use of A/A_o (or N/N_o) makes the comparison easy. Plotting this ratio instead of raw activity, A, normalizes the data and all plots are scaled the same on the y-axis and start with a y-intercept of $A/A_o = 1$.

When the half-life is very large for an isotope the graph appears to look linear in nature as shown below. The linear regression results are very good as r^2 is very close to one; however, the data is exponential (analysis of residuals will show a pattern).
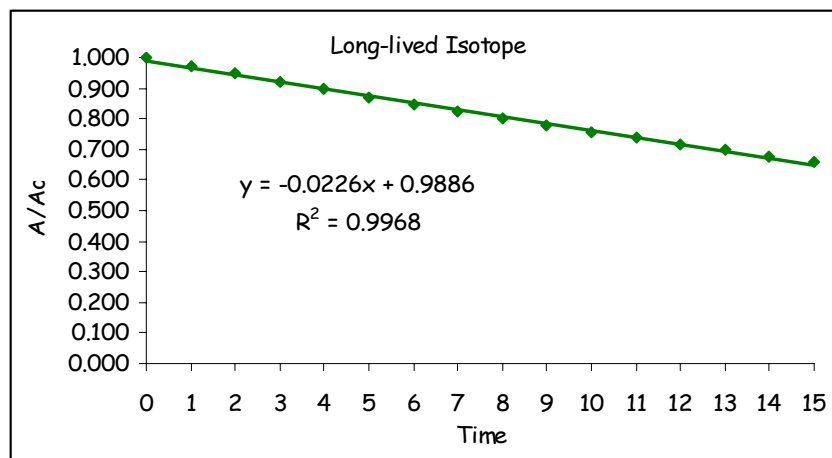
Early work with long-lived isotopes did, initially, give scientists the impression that decay was linear.

For the logarithmic plots, the half-life is determined as illustrated on the graphs given below.
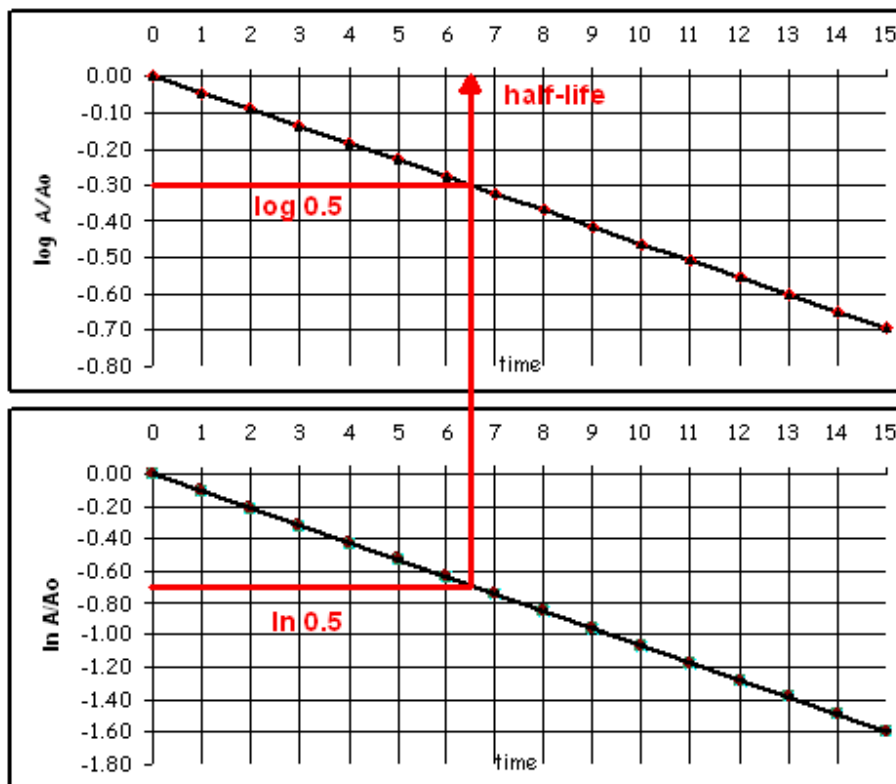
These linear plots are generated from the logarithmic equations given below, written in the form of y = (m)x, where the slope is given in parentheses:

$$\log \frac{N}{N_o} = \left(\frac{-k}{2.303}\right)t \qquad \ln \frac{N}{N_o} = (-k)t$$

The half-life on each plot is determined graphically where N/N_o = 0.5 or log N/N_o = 0.301 or ln N/N_o = 0.693.   Usually the better choice for hand-drawn data analysis is the linear logarithmic plots.   Linear fits using a straight edge are more accurate than a sketch of a curved exponential fit.

Let's consider how the daughter behaves as the parent decays. Click on the daughter tab.

9.   How would you describe the behavior of the daughter product compared to the parent?

10.   Notice the behavior of the P + D line on the graph. Why does $P + D = P_o$?

11.   If the only source of the daughter isotope is from the decay of the parent isotope, then the amount of each can be used to determine the age of the sample. This is the basis of radioactive dating in geology. When igneous rocks form from a melt, the initial amount of parent is set at time equal to zero. Usually the D/P ratio can be determined by mass spectrometry. How does the D/P ratio behave as a sample gets older?

12.   How is the D/P ratio related to the P/P_o, which is the same as A/A_o?

13.   How would the behavior of the daughter isotope behave if it was radioactive too? Sketch your prediction of the behavior below.
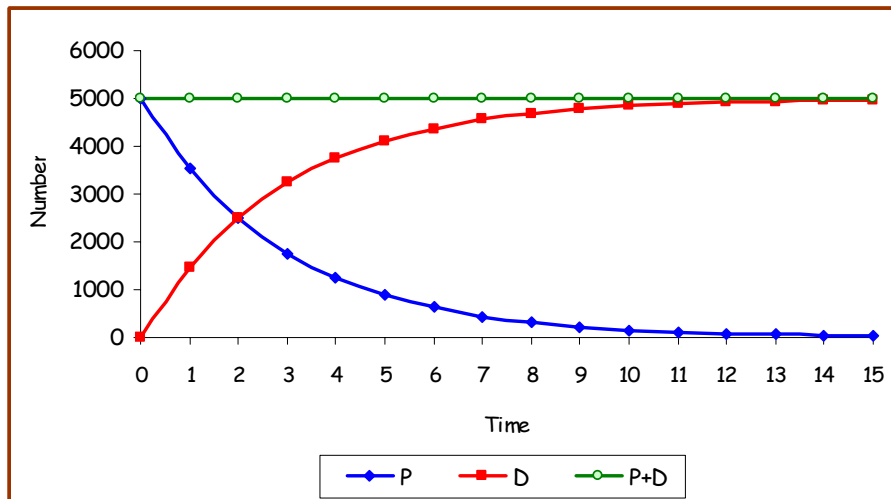
The daughter isotope grows in at a one-to-one rate ($N_D = N_o - N_P = N_o - N_o e^{-kt}$) as the parent isotope decays ($N_P = N_o e^{-kt}$). At any time the sum of the parent and the

daughter isotopes (P + D) must equal the starting parent amount, P_o. The P = D point (intersection of the two curves) occurs at the half-life or when P_o = 0.5.

For the D/P ratio from mass spectrometry and the rules of logarithms, the red box shows how the ratio is related to the age, t, of the sample.

$$\ln\frac{P}{P_o} = \ln\frac{P}{P+D} = -kt \qquad\Longrightarrow\qquad \ln\frac{P_o}{P} = \ln\frac{P+D}{P} = \boxed{\ln\left(1+\frac{D}{P}\right) = kt}$$

Now click on the unstable daughter tab to explore the situation where $P \rightarrow D \rightarrow S$.

14.   How accurate was your prediction in question 13 do?

15.   How does the sum of P + D + S compare to the starting amount of parent, P_o?

16.   Let's compare the behavior of this system for the following half-lives. To do this, click on the total rad tab to see total radioactivity given by P + D on this plot. Enter the values in the table below. Record a sketch of each set of resulting plots for comparison.

$t_{1/2}$ P	$t_{1/2}$ D	What controls radioactivity?
1	25	
25	1	
5	5	

17.   If you were measuring a D/P ratio to determine the age of a sample and the daughter was radioactive, how would it influence the determination of the age? Explain the error.

When the daughter isotope is radioactive or unstable, it must decay along with the parent isotope. The stable isotope grows in depending on the rate of the longer-lived isotope decay. Total radioactivity, P + D, is governed by the longer half-life isotope. The slower reaction (smaller rate constant or larger half-life) controls the overall rate in consecutive reactions (A $\rightarrow$ B $\rightarrow$ C). The P + D + S = P_o and if all the radioactive material decays then S = P_o or all stable isotope exists.

If the daughter decays, the D/P ratio will be lower and produce an error in the age determination. Since the D/P increases with age, the error will give a younger age. The daughter isotope must be stable for the use of the D/P ratio to work in age determinations.

The last two tabs deal with experimental error involved with the measurement of radioactivity or counting error. Radioactive decay is measured by counting the decay events with a device such as a Geiger counter. Before you proceed be sure the Analysis ToolPak is loaded in Excel. Go to the Tools menu, select Add-Ins… and then select the Analysis ToolPak box and click OK. Now click on the counting error tab.

18.   Vary the slider to increase the amount of counting error in the measurements. How does counting error influence the regression equation for the decay constant?

19.   Where does the counting error, as percent error, have the greatest impact?

20.   What happens to counting error if the starting activity is decreased? Pay attention to the scale values on the % error axis.

The counting error, as percent error, grows larger as the activity drops, so it is worse at low activities at the later times in the measurement. This is a **random error** since absolute error or $A_{measured} - A_{theoretical}$ against time plots randomly as shown on the graph below.
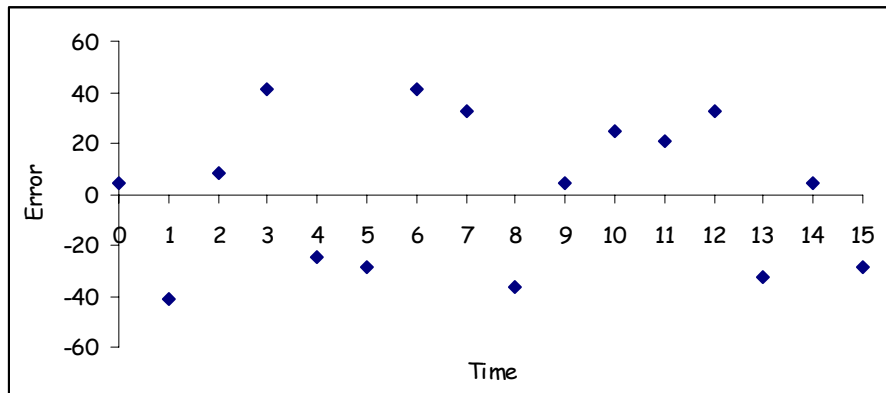
This random variation in the counts becomes a serious problem when the total counts are a small number such as at later times. Larger percent errors can greatly influence the exponential regression results. One method to minimize counting error is to increase the amount of starting activity (larger P_o). Alternatively the regression can be recomputed on the earlier data points to improve results by eliminating the points that approach background levels with their large percent errors. Examining the goodness of fit is important here.

Now let's add background in with the counting error. Click on the background tab. Background radiation is the counting of events from the decay of natural or man-made radioactive isotopes in the environment and is <u>usually constant over time</u> in the laboratory. The background level can be changed and the background correction slider adjuster to correct for background radiation.

21.   How does not correcting for background radiation influence the results of a decay curve measurement (compare the red and blue data and curves)?

22.   Where does the background have the greatest impact?

23.   How is background radiation corrected?
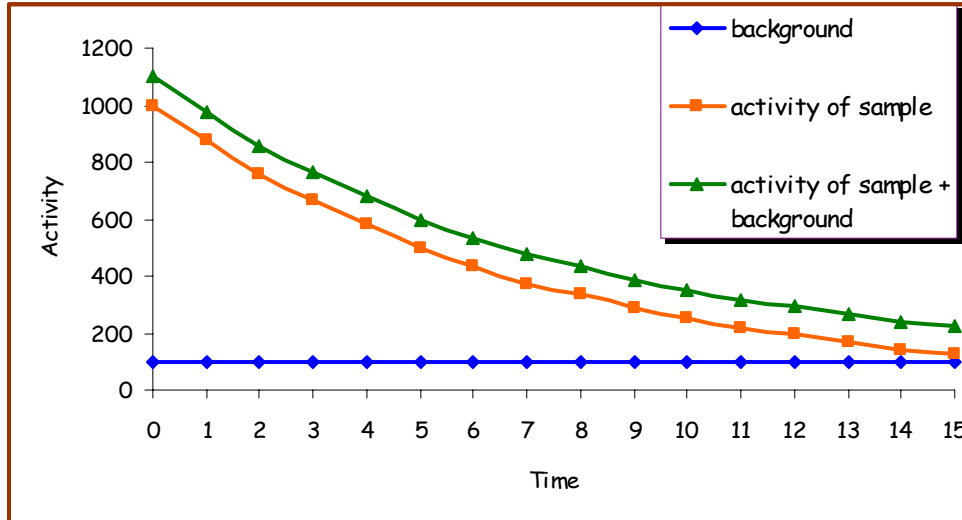
24.   Does increasing the starting activity help or hinder the error from background radiation?

25.   Find five different sources of background radiation in the environment that are due to radioactive materials. List the isotope, its half-life, the source, and typical dose.

Isotope	Half-life	Natural or man-made?	Source	Dose

Cite your reference source.

Background radiation has the largest effect at low activities and has a major influence on the resulting exponential regression. On a graph, the addition of the background causes a translation along the y-axis for raw activity data compared to the background corrected data.

In the actual analysis the A_o value has the background included when you normalize the data and the regression line appears to rotate in a counter-clockwise direction. The following changes occur in the regression results:

- r^2 value decreases (should be doing an exponential regression that can handle the translation of the form - $y = ae^{-kt} + c$);
- decay constant, k, is altered to a lower incorrect value (error of a higher half-life would result), and;
- for $y = ae^{-kt}$ notice that the value of "a" is now less than one (notice a = 1 or very near one for background corrected data).

A **systematic error** is induced by the lack of background correction (a constant factor is added causing a bias in the values). Background radiation, which is constant over time, is subtracted from the sample activity for all measurements, $A_{raw} - A_{background}$. Increasing the starting amount of parent may decrease the effect of the background (becomes a smaller percent error); however, it does not correct for it. Background radiation must be corrected for to get good results!

Now try the assessment questions that follow on the next two pages.

Assessment

1.   Using the raw data given below and the background radiation of 225 cpm, complete the table below, plot hand-drawn A/A_o against time, and log or ln A/A_o against time graphs on graph paper. Attach both graphs to this assignment. (cpm = counts per minute)

| Time, minutes	Raw parent activity, cpm	Background corrected activity, cpm	A/A_o	ln or log A/A_o
0	1445			
1	1300			
2	1137			
3	1030			
4	939			
5	835			
6	743			
7	674			
8	641			
9	575			
10	530			
11	504			
12	456			
13	413			
14	400			
15	364			

2.   On the A/A_o against time graph, sketch a smooth curve and estimate the half-life on the graph. Show on the graph!

3.   Perform a linear regression on the logarithmic graph and report the results including the goodness of fit. From the regression results on the logarithmic graph determine the half-life. Describe how you determined the half-life.

4.   How do the two graphical methods compare? Describe a possible cause of any difference. Can you do anything to improve either method? Explain.

5.   For the data above in question 1, we assumed the daughter was stable and the raw activity measured was only for the parent isotope. Suppose the daughter had a half-life of 400 minutes. Would this influence your results above? Why or why not? If yes, explain how.

6.   Which isotope below on the graph would be more suitable for use in a nuclear medicine test, where you do not want the isotope to maintain a long presence in the human body? Explain your choice. Assume the time scale is given in days.
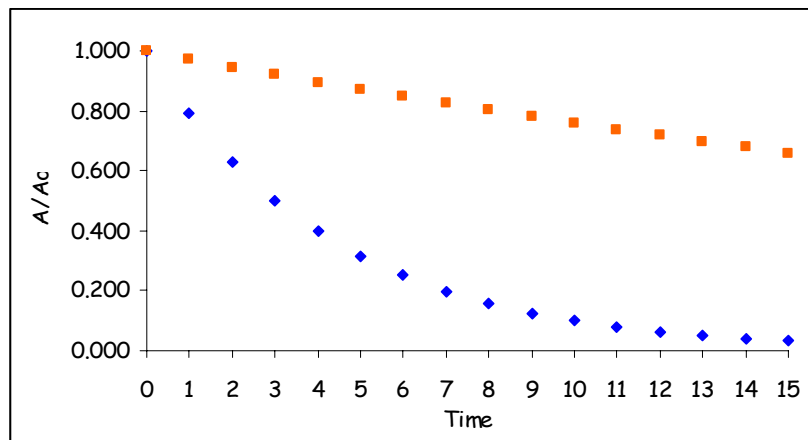
7.   Use the safe tab (far right) on the Excel spreadsheet to determine when radiation levels are considered safe based on units of half-lives. How long is this for your answer to question 6?