CoreModels

Flow Rates

A CORE LEARNING GOALS ACTIVITY FOR SCIENCE AND MATHEMATICS

Summary
Students use proportional reasoning and data analysis to build a computer model to explore the dynamic relationships in a model of rates of flow into and out of a container. This model has many applications across a variety of disciplines.

This material is based on work supported by the National Science Foundation. Any opinions, findings, conclusions, or recommendations expressed on these pages are those of the CoreModels Project of the Maryland Virtual High School of Science and Mathematics and do not necessarily reflect the views of the National Science Foundation.

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Date last modified: September, 2000

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LESSON OVERVIEW

<u>Introduction</u>

This lesson/model is highly recommended both for math teachers and for science teachers in all fields. As will be seen below, the lesson can be used effectively for a variety of content applications, and at various points in a course sequence. One reason for the lesson's power is its wide range of applications, as shown by the following examples of analogous concepts which the model serves to illuminate:

- A lake or pond, fed by one river or stream, and drained by another (additional feed: rain)(additional drain: percolation to water table)
- An individual cell's water balance, whether it is a plant cell trying to maintain turgor, or a protist trying to pump water out to counter osmotic pressure
- A specific organ or cavity's normal range of hydrostatic pressure (e.g., urine in a bladder, or cerebrospinal fluid in a mammalian dorsal cavity)
- An organism's overall fluid balance
- Kinetics, relative reaction rates, and dynamic equilibrium in chemical reactions
- Thermal content of a solid body
- Nuclear binding energy, fed by particle bombardment and drained by radioactive decay
- How will the time needed to fill your bathtub be affected by an ill-fitting drain stopper?
- How long will it take you to go broke if you spend more than you earn? (and pay 18% extra for everything as well)
- What kind of function has a rate of change which is proportional to itself?

Another reason for the **Flow Rate** lesson's power is the students' *very direct mental mapping* between concrete observations of the demonstration and mathematical modeling in the software. Not only is the symbol system of the software a close mimic of the apparatus, but there is also the powerful advantage of *real-time data analysis* -- an advantage well-known from CBL lab experience in teaching graphing. With mathematical modeling, this real-time advantage for teaching and learning math-science symbolism is taken to an even deeper cognitive level.

<u>Teaching objectives</u>

(a) to develop students' proportional reasoning skills
(b) to develop students' skills at building mathematical models
(c) to refresh students' ability to derive linear equations from plots of apparently linear phenomena
(d) to refresh students' manual graphing abilities
(e) to introduce students to the use of computerized output graphs, and their advantages over manual techniques, including real-time data display and analysis.
(f) to establish the properties of a basic inflow-reservoir-outflow model as the conceptual template underlying a variety of natural and social processes.

LESSON PLAN

MATERIALS:

- class set of student worksheets
- overhead transparencies of student worksheets
- STELLA software
- flow rate demo set-up :
  glassware (largest available separatory funnel, 50-ml graduated buret, and graduated flask); large ring stand; 2 rings; clamps; supply of brightly colored water

PREP TIME: 1 hour

CLASS TIME: 70-90 minutes

Flow Model - Teacher Notes

In the process of doing this activity, the students will discover that the outflow rate is not constant. For students who are not yet in Algebra II, we will assume that the relationship between the outflow rate and the buret volume is linear. Those students should follow the activity as it is written. For more advanced students, you may wish to introduce the ideas explained below.

1. Is the relationship of the outflow rate to the burette volume really linear?
   NO

2. What is the true relationship?
   outflow rate = k*sqrt(volume)

3. Why do the points look like they are on a line?
   The curvature is not noticeable until the burette is almost empty, and collecting data for that condition is very difficult.

4. If my students do not know how to curve-fit a power function (y = a*x^0.5), is it acceptable to continue to use the linear equation (y = mx + b)?
   YES, if you emphasize that the line is only an estimate of the relationship. Since the point of the lab is the existence of a dependency between the outflow rate and the burette volume, the line is adequate.

5. How do you know that the function is a power function?

Let's look for some theoretical background on this problem. The potential energy of the system at any point in time is mass*gravity*height or mgh. The kinetic energy transported out of the burette at any point in time is 1/2*mass*velocity^2 or 1/2mv^2.

Assuming all of the potential energy is converted into kinetic energy (conservation of energy), set $1/2mv^2 = mgh$. Simplify to $v^2 = 2gh$, where velocity is in m/s, gravity is in m/s^2 and height is in m. Since we are using mL and minutes, the appropriate conversions must be made.

$v^2 = 2gh$	(m/s)
v = sqrt(2*9.8*h)	(m/s)
v = sqrt(2*980*h)	(cm/s)
v = 60*sqrt(2*980*h)	(cm/min)

Assume the burette is a perfect cylinder volume = $\pi r^2 h$

πr^2*v = πr^2*60*sqrt(2*980*h)	(cm^3/min)
vel = πr^2*60*sqrt(2*980*h)	(mL/min)
vel = 2656*πr^2*sqrt(volume/πr^2)	(mL/min)
vel= 2656*sqrt(πr^2)*sqrt(volume)	(mL/min)
vel = 4708*r*sqrt(volume)	(mL/min)

This shows that the outflow (velocity) is proportional to the square root of the burette volume.

Sample Data

Buret vol (ml) 50	Elapsed Time (s) 0	Volume Change (ml)	Time Change (s)	Avg. Outflow Rate (mL/s)	Avg. Outflow Rate (mL/min)	Buret Volume Midpoint (mL)
45	35	5	35	0.143	8.571	47.5
40	75	5	40	0.125	7.500	42.5
35	124	5	49	0.102	6.122	37.5
30	179	5	55	0.091	5.455	32.5
25	244	5	65	0.077	4.615	27.5
20	306	5	62	0.081	4.839	22.5
15	381	5	75	0.067	4.000	17.5
10	470	5	89	0.056	3.371	12.5
5	575	5	105	0.048	2.857	7.5

At first glance, there appears to be a linear relationship. The equation of a line can be found in a number of ways. Those students with a graphing calculator or familiar with a program like Graphical Analysis can perform linear regression. Using the sample data, the equation y (outflow in mL/min) = 0.134x (burette volume in mL) + 1.58 has a mean square error of 0.165 - that seems like an excellent fit.

Note: Students not having access to regression analysis tools may determine a linear equation by sketching a line that is close to most data points. Using the estimated coordinates of the endpoints (x_1, y_1) and (x_2, y_2) of the line, the student can find m, the slope of the line, by using $m=(y_1-y_2)/(x_1-x_2)$. Using m and (x_1,y_1) the student can substitute into $y_1 = m*x_1 + b$ to find b, the y-intercept. With m and b, the equation is y = mx+b. The mean square error is a bit tedious to perform by hand - for every data point, the difference between its y-coordinate and the corresponding y-coordinate on the line must be calculated and squared. Sum those squares and divide by the number of points and you will have the mean square error.

In either case, if we look more carefully, we are alarmed that the outflow does not approach zero as the burette empties. At volume = 0, the outflow would be 1.58 mL/min. Is this just a result of error in data collection?

Using a graphing calculator or Graphical Analysis (or similar software), choose the power function y = a*x^b. With the sample data, y = 0.828*x^0.565 with a mean square error of 0.324. It is reassuring to see that the coefficient of the power function is close to 0.5 (i.e. square root). Next, try a power fit with a fixed exponent of 0.5. This results in y = 1.05*x^0.5 with a mean square error of 0.408.

Why do we get a larger error here than with the linear regression? It appears that we should collect more data. If we continue the experiment so that the burette becomes nearly empty and if we start measuring the time for one mL decrements rather than 5 mL ones, we will see the sharp downward trend in the outflow rate as we near zero volume. If we then do both the linear and power curve fits, we will see a smaller error for the power function. Of course, this is still a simplification of the problem. We have ignored forces such as friction, but the influence of those forces would only impact the coefficient of the function, not the exponent.

It is important to note that the data alone were insufficient to determine the best curve fit. Actually, the ‘best’ answer (power) turned out to have more error than another answer (linear) that was discarded. How do we explain this seeming contradiction to our students? This is an excellent opportunity to point out the need for an extensive set of data points and the importance of the extremities of the data set. This example also reinforces the need to look to scientific theory for guidance.

If your students still insist that the best fit is the function with the least error, have them try y = a*x^b + c with the sample data. The resulting function is y = 0.00336*x^1.92 + 2.96 with a mean square error of 0.0751. Based on error alone, this appears to be a better fit than the line was. But, just as with the linear fit, the outflow rate does not approach zero as the burette empties! Clearly, this function is not the answer.