CoreModels

Universal Gravitation

A Core Learning Goals Activity for Science and Mathematics

Summary: Students construct a STELLA model to demonstrate the effect of universal gravitation between two objects. An extension to Coulomb's Law is available for use when students learn about electromagnetism.

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Date last modified: October, 1998

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Universal Gravitation

Teacher Guide

Overview:

This model shows the effect of gravitational attraction between two objects. The force between them causes an acceleration, which then causes the objects to move toward each other.

Activities at a Glance:

Activity	Title	Materials	Estimated Time
1	Earth Gravity Exploration	Pasco's ball drop apparatus	45 min.
2	Building a Model for Universal Gravitation	STELLA, computers	45 min.
3	Extension: Earth Gravity	STELLA, computers	45 min.
4	Extension: Building a Model for Coulomb's Law	STELLA, computers	45 min.

Prior Knowledge:

The student should be familiar with the Universal Gravitation equation and should have built some simple STELLA models involving force as well as a model involving more than one particle. The student should recognize that the two particles will need to be placed at different positions to start and that gravity is an attractive force. In order to use the extension activity, the students should be aware of the relative strengths of the gravitational and electrostatic forces. These strengths have implications for the models as explained below.

Best Practices:

To start the discussion of Universal Gravitation, the teacher should engage the class in some real life activities. For example, have a student push a table. Identify how force acts on mass to accelerate it. Ask what stops the table from accelerating beyond a certain speed. Drop a book. Why does the book accelerate? Note that the Earth is acting on the book without the two bodies being in contact.

Activity 1 serves as an exploration and explanation activity in which the students calculate the value of the Earth's gravity. In Activity 2, the students build a STELLA model to extend their understanding of the kinematics and forces involved in the motion. The Teacher Guide contains detailed notes

regarding the construction of the model.

In activity 3, the students revise the Universal Gravitation model to show Earth's gravitational acceleration.

The Extension Activity may be used later in the year after the students have studied the basic laws of electromagnetics. With minor modifications, the students turn their Universal Gravitation model into a Coulomb's Law model.

Science Core Learning Goals Match:

Core Goal 1: Skills and Processes

4.   Expectation: The student will demonstrate that data analysis is a vital aspect of the process of scientific inquiry and communication.

   (8)   The student will use models and computer simulations to represent
          systems.

6.   Expectation: The student will use mathematical processes.

   (2)   The student will express and compare small and large quantities
          using scientific notation and relative order of magnitude.

   (3)   The student will manipulate quantities and/or numerical values in
          algebraic equations.

Core Goal 5: Concepts of Physics

1.   Expectation: The student will know and apply the laws of mechanics to explain the behavior of the physical world.

   (3)   The student will analyze and explain how changes in an object's
          motion are described by Newton's Laws.

   (4)   The student will analyze the behavior of forces.

          At least --
               -- Comparison of relative magnitude
               -- The inverse square nature of gravitational and
                    electromagnetic forces

2.   Expectation: The student will know and apply the laws of electricity and magnetism and explain their significant role in nature and technology.

   (1)   The student will describe the types of electric charges and the
          forces that exist between them.

          At least --
               -- Coulomb's Law

Universal Gravitation
Teacher Guide

Activity 1 - Earth Gravity Exploration

Using Pasco's ball drop apparatus and various size steel ball bearings, determine the acceleration due to the Earth's gravity: **g**. Find the mass and diameter of each sphere. Place each sphere so the bottom of the sphere is the same distance above the pedal and drop each sphere at least 5 times.

Using the average time of the drops for a given sphere, calculate the average value of **g**.

Q1.1: Does the mass or the diameter of the sphere make any difference in the value of **g**? If so, what is the trend?

> **The mass of the sphere should make no difference in the value of g. The Diameter of the sphere might appear to make a difference if the student does not measure accurately from the bottom of the sphere to the floor for each trial. Taking one measurement and then changing the diameter changes the actual distance dropped.**

Now vary the height for a given size sphere. Make a table for your data. Use at least 4 different heights and run at least 5 trials at each height. Again, calculate the average time for each height and calculate the average value of **g**.

Q1.2: Does the height make any difference in the acceleration? If so, what is the trend?

> **The height makes no difference in the acceleration. It does make a difference in the final speed. (Emphasize the differences among final speed, average speed, and acceleration.)**

Q1.3: What part of the equation $F = \dfrac{G\, m_1 m_2}{r^2}$ equals **g**   if

$m_1 = m_{earth}$ and $r = r_{earth}$?

> **g = G m1/r2**

Q1.4: What is the value and the units for **G**?

> **G = 6.67 E-11 Nm2/kg2**

Activity 2 - Building a Model for Universal Gravitation

Notes to the Teacher:

The student could start with a basic force model, from scratch or from the action-reaction model. Instructions are included for the first and last options. The student needs to build two sets of icons, one for each particle. The distance icons would more properly be labeled position in this model. The student needs to determine a relative point to measure from and then assign each particle an appropriate value for the initial position. Students generally over-estimate the size of the gravitational force, so they need to be reminded of its relative strength when determining the initial mass and distances. If the particles are too small or too far apart, then the force won't be large enough to cause any appreciable movement while the model runs. Also students need to recognize that this is an attractive force, so the accelerations must have opposite signs if the two particles are to move toward each other.

Appropriate time scales should be chosen. There is no fail-safe way in the model to tell the particles that they have collided, so they will fly past each other. Students will only see this for certain selections of time, mass and distance. Again, this is an excellent opportunity to teach the limtations of modeling, i.e. that the model can only predict for conditions which have been programmed in.

In creating a position graph, showing the two particles moving toward each other, students will need to set the axes so that there is only one set of values for both axes. It is also useful to create an x-y scatter graph of the force vs. the distance between the particles. Again, if the axes are adjusted, this graph shows the expected inverse-square curve.

Annotated Student Guide:

Based on your knowledge of force models, it should be fairly simple to build a model which shows how two objects will move toward each other due to gravitational forces. This model is different from earlier models in that you have two objects, so each object will need its own set of distance, velocity and acceleration icons. Also, since we are interested in the position of each object, the distance stocks should be re-named as position stocks. Also, the net force icon will reflect the fact that the two masses are interacting.

1. Open STELLA.

2. Create a series of stocks, flows and converters to represent position, rate of change of position, velocity, rate of change of velocity, acceleration and mass. You will create two sets of these, one for mass 1 and one for mass 2. All flows should go **into** stocks. Each mass should connect to its acceleration (mass 1 to acceleration 1, mass 2 to acceleration 2). Just to be safe, make sure your velocities can be negative and that your flows are biflows. **(See Appendix A)**

3. Create one net force converter. This converter should have connectors going to each of the acceleration converters. Connectors from each of the masses should lead to net force and connectors from each of the distance stocks should lead to net force.

Q2.1: Will this net force be constant or will it change? Explain.

> **The gravitational force will be directly proportional
> to the mass of each body but inversely proportional
> to the square of the distance between the bodies. So,
> the force will increase while the distance between
> the bodies decreases.**

4. All equations should be as before except for the net force equation. This means that you know what to enter for rate of change of position, rate of change of velocity and acceleration. The initial velocities of both particles will be zero. The initial values for position and the values for mass are discussed below. **(See Appendix A)**

5. If all the right connections are made, you should be able to enter the equation for universal gravitation into the net force converter. It should look something like this

        6.67E-11*mass1*mass2/(Distance1 - Distance2)^2

Q2.2: The equation in the book just says d^2. Why are we subtracting the distances before squaring?

> **d^2 in the book means the distance between the centers
> of each body. On a distance time graph that would be
> the difference in distance between the temporal
> positions of each body squared.**

6. Now comes the experimental part. The initial position for mass 1 should probably be 0. That gives us a reference point. The initial position for mass 2 should be somewhere farther away. Try 0.2 (that would be 20 cm). Remember that the gravitational force is very small. Now make the masses large enough (something E5 or in the 100,000's). Create a graph showing the position of each mass. You should set the axes to have the same scale (0 to your position for mass2). Run the model and sketch the graph.
(See Appendix B)

Q2.3: Is the graph what you expected? Explain.

> **The graph did not curve very much nor did the objects
> meet in the time frame of the graph. The two distances
> stayed pretty much the same over the time frame of the
> run. Either the masses must be increased considerably
> or the dt increments increased.**

7. Sketch the model on your paper as well. We will build a similar model when we look at electromagnetic forces. Be sure to put your initial positions and masses on the sketch for comparison.

8. Create a second graph. This will be a scatter graph. Put distance_between on the x axis and force on the y axis. Run the model again

and sketch the graph. **(See Appendix C)**

Q2.4: What is the relationship shown by this graph?

Putting distance on the x axis and net force on the y axis, creates a graph that looks almost linear with a negative slope. Closer inspection reveals a curve. Changing the masses to the e6 range makes the curve more pronounced. The curve suggests the inverse square law is operating.

Experimentation will show that a great curve is achieved by setting the masses = 10^5, the initial position for the second mass to 0.12, running the model for 12 sec with a dt of 0.01 and Euler's.

9. Return to the first graph (looking at the two positions). Experiment with different initial positions and masses.

Q2.5: What happens if the masses are too large?

If the masses are too large in the range of 6 e5 and the time range is increased to 4 seconds, the masses go right through each other after they meet.

Q2.6: What happens if the distances are too large?

If the distances are too large, the graph stays rather flat and difficult to see the effect of the inverse square law.

10. Save your model as Universal Gravitation.

Activity 2 (Alternate) - Building a Model for Universal Gravitation from the Action-Reaction Model

Building a Universal Gravitation model is simple if you start with the Action-Reaction model. There are afew significant changes such as renaming the distance stocks as position, creating a distance converter and changing the applied force icon so that it will reflect the fact that the two masses are interacting.

1. Open the **Force 2 Objects** model.

2. Change the name of the distance stocks to position and position 2.

3. Create a distance converter. This converter should calculate the difference between the two positions. Later on you will be told to start position2 at a larger number than position1, so set the formula in the distance converter so that the distance will be positive. **Note: a negative distance won't affect the first graph because distance is squared, but the second graph between applied force and distance will curve backwards.**

3. The applied force converter will hold the equation for Universal Gravitation.

Q2A.1: Before you enter the equation into this converter, what other converters will need to be connected to it?

> **Both mass converters and the distance converter should be linked to the applied force converter.**

*Q2A.2: Will this applied force be constant or will it change? Explain.

> **The gravitational force will be directly proportional to the mass of each body but inversely proportional to the square of the distance between the bodies. So, the force will increase while the distance between the bodies decreases.**

4. Make the connections you suggested in Q2.1 above and then enter the Universal Gravitation formula (using STELLA names) into the applied force converter.

Q2A.3: Record here the equation as it appears in your converter.

> **Student answers will vary dependent on the names they have used for icons, but should be the appropriate equation.**

5. Now comes the experimental part. The initial position for mass 1 should probably be 0. That gives us a reference point. The initial position for mass 2 should be somewhere farther away. Try 0.2 (that would be 20 cm).

Remember that the gravitational force is very small. Now make the masses large enough (something E5 or in the 100,000's). Create a graph showing the position of each mass. You should set the axes to have the same scale (0 to your position for mass2). Run the model and sketch the graph.

Q2A.4: Is the graph what you expected? Explain.

> **The graph did not curve very much nor did the objects meet in the time frame of the graph. The two distances stayed pretty much the same over the time frame of the run. Either the masses must be increased considerably or the dt increments increased.**

6. Create a second graph. This will be a scatter graph. Put distance on the x axis and applied force on the y axis. Run the model again.

*Q2.5: What is the relationship shown by this graph?

> **Putting distance on the x axis and net force on the y axis, creates a graph that looks almost linear with a negative slope. Closer inspection reveals a curve. Changing the masses to the e6 range makes the curve more pronounced. The curve suggests the inverse square law is operating.**
>
> **Experimentation will show that a great curve is achieved by setting the masses = 10^5, the initial position for the second mass to 0.12, running the model for 12 sec with a dt of 0.01 and Euler's.**

7. Return to the first graph (looking at the two positions). Experiment with different initial positions and masses.

Q2.6: What happens if the masses are too large?

> **If the masses are too large in the range of 6 e5 and the time range is increased to 4 seconds, the masses go right through each other after they meet.**

Q2.7: What happens if the distances are too large?

> **If the distances are too large, the graph stays rather flat and difficult to see the effect of the inverse square law.**

8. Save your model as Universal Gravitation.

Activity 3: Universal Gravitation Model and the gravitational acceleration of Earth

Once you have a working Universal Gravitation model that has been checked by your instructor, modify the model to so that it represents the attraction between Earth and an object 1500 m above Earth.

Information needed:

Mass of Earth: 5.98×10^{24} kg

Radius of Earth: 6.38×10^{6} m

Mass of object: 1 kg

Reminder: The distance in the Universal Gravitation equation is from center to center of the two objects. This was not explicitly part of the earlier models but becomes a crucial point in this one.

Q3.1: If the initial position of Earth is at 0, what is the initial position of an object 1500 m above Earth?

$$\boxed{6.38 \times 10^{6} + 1500 \text{ m}}$$

After you have a working model:

Q3.2: What is the average acceleration of the object? $\boxed{-9.8}$ Of Earth?

$$\boxed{1.64 \times 10^{-24}}$$

Q3.3: Look at the position graph and comment on the motion of the Earth.

> **If the axes are not locked for a preset value, the graph will show that there is some movement (very small) of Earth.**

Q3.4: Vary the mass of the object from 1 kg to 1000 kg and observe the acceleration of the object. How does mass of a falling object affect the value of the gravitational acceleration?

> **It doesn't have any noticeable affect. This point should be driven home and the connection made to the free fall model, where mass does not affect the rate of acceleration.**

Appendix A
Model for Universal Gravitation

Diagram
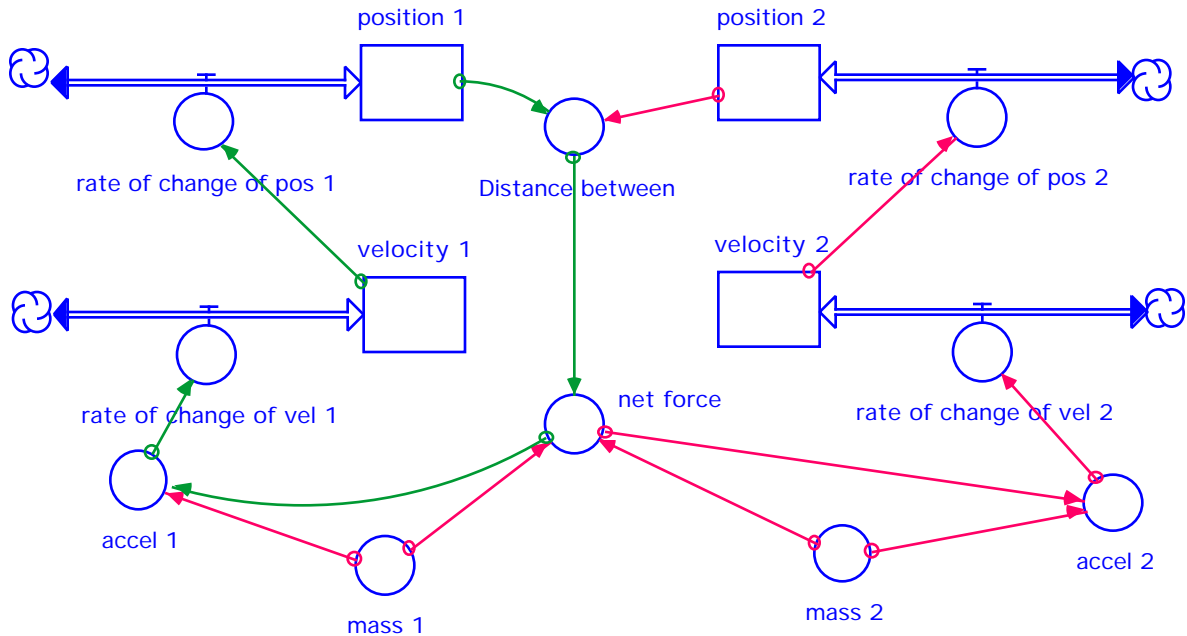
Equations

```
position_1(t) = position_1(t - dt) + (rate_of_change_of_pos_1) * dt
INIT position_1 = 0
rate_of_change_of_pos_1 = velocity_1
position_2(t) = position_2(t - dt) + (rate_of_change_of_pos_2) * dt
INIT position_2 = .2
rate_of_change_of_pos_2 = velocity_2
velocity_1(t) = velocity_1(t - dt) + (rate_of_change_of_vel_1) * dt
INIT velocity_1 = 0
rate_of_change_of_vel_1 = accel_1
velocity_2(t) = velocity_2(t - dt) + (rate_of_change_of_vel_2) * dt
INIT velocity_2 = 0
rate_of_change_of_vel_2 = accel_2
accel_1 = net_force/mass_1
accel_2 = -net_force/mass_2
Distance_between = position_2-position_1
mass_1 = 6E6
mass_2 = 6E6
net_force = 6.67E-11*(mass_1*mass_2)/(Distance_between)^2
```
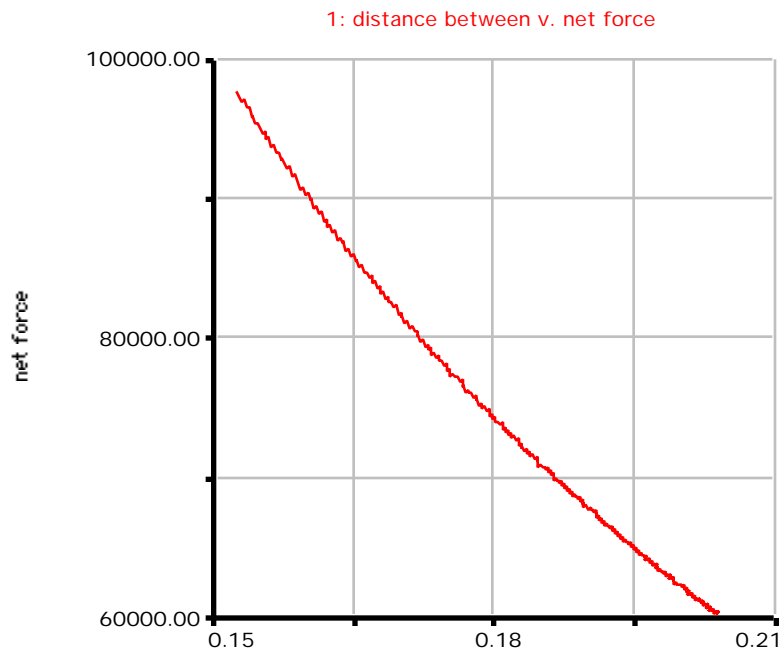
Time Specs

```
Range: 0 -2, dt = 0.1,   Integration Method: Runge-Kutta 2
```

Appendix B
Position vs. Time - Graph

Graph 1: p1 (1:1 MASS RATIO)   Time   1:39 PM   6/16/98
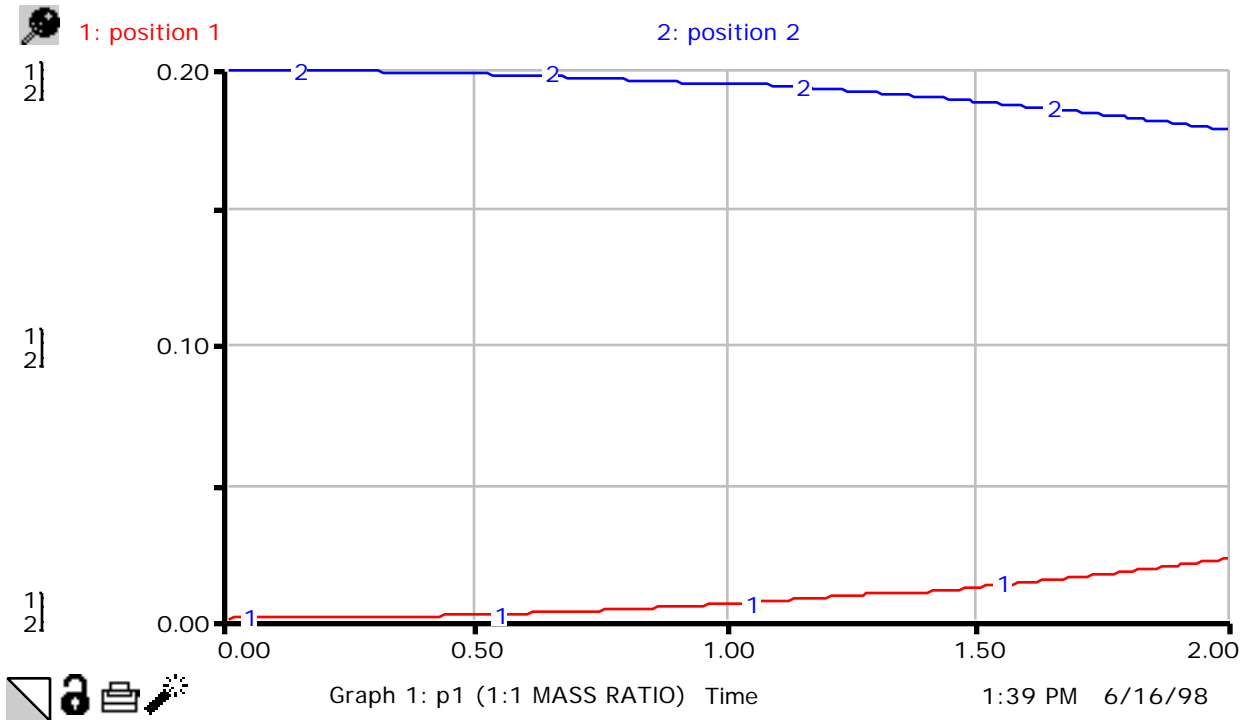
Appendix C
Distance Between vs. Net Force - Scatter Graph

1: distance between v. net force
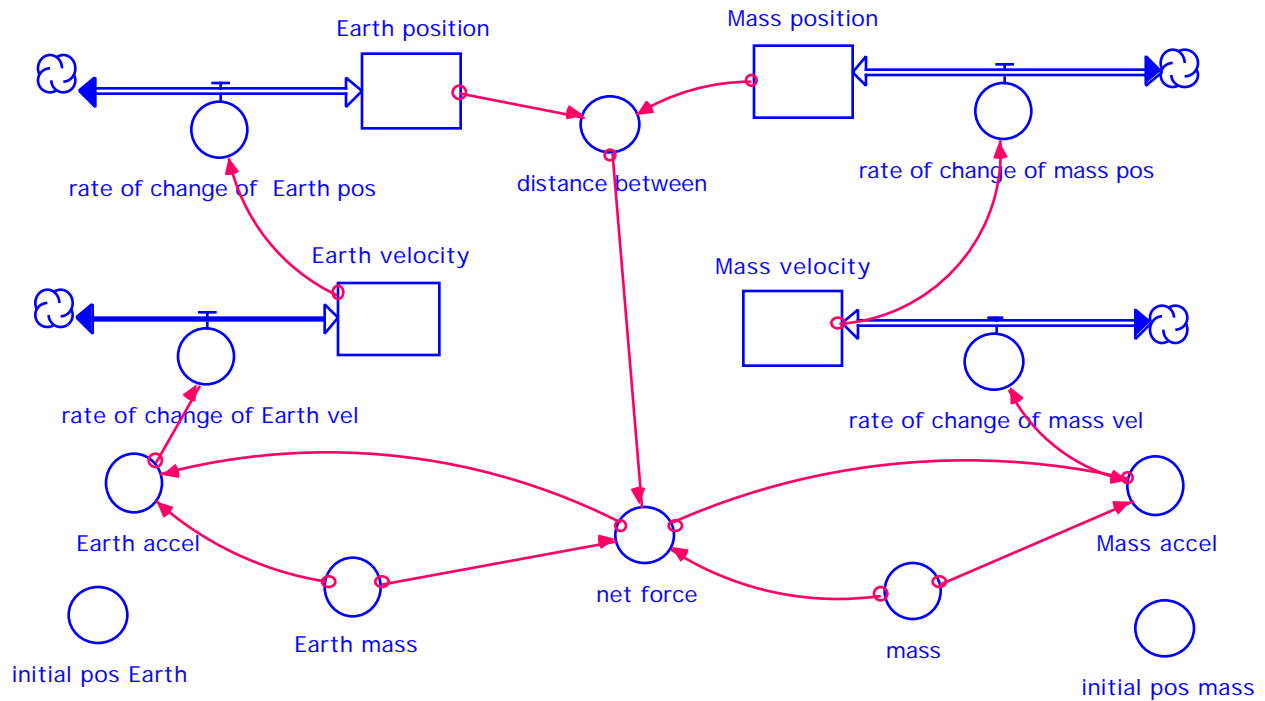

Graph 2 (Untitled)   distance between   1:50 PM   6/16/98

Appendix D
Earth Gravity Model


```
Earth_position(t) = Earth_position(t - dt) + (rate_of_change_of__Earth_pos) *
dt
INIT Earth_position = initial_pos_Earth

INFLOWS:
rate_of_change_of__Earth_pos = Earth_velocity
Earth_velocity(t) = Earth_velocity(t - dt) + (rate_of_change_of_Earth_vel) *
dt
INIT Earth_velocity = 0

INFLOWS:
rate_of_change_of_Earth_vel = Earth_accel
Mass_position(t) = Mass_position(t - dt) + (rate_of_change_of_mass_pos) * dt
INIT Mass_position = initial_pos_mass

INFLOWS:
rate_of_change_of_mass_pos = Mass_velocity
Mass_velocity(t) = Mass_velocity(t - dt) + (rate_of_change_of_mass_vel) * dt
INIT Mass_velocity = 0

INFLOWS:
rate_of_change_of_mass_vel = Mass_accel
distance_between = ABS(Mass_position-Earth_position)
Earth_accel = net_force/Earth_mass
Earth_mass = 5.98E24
```
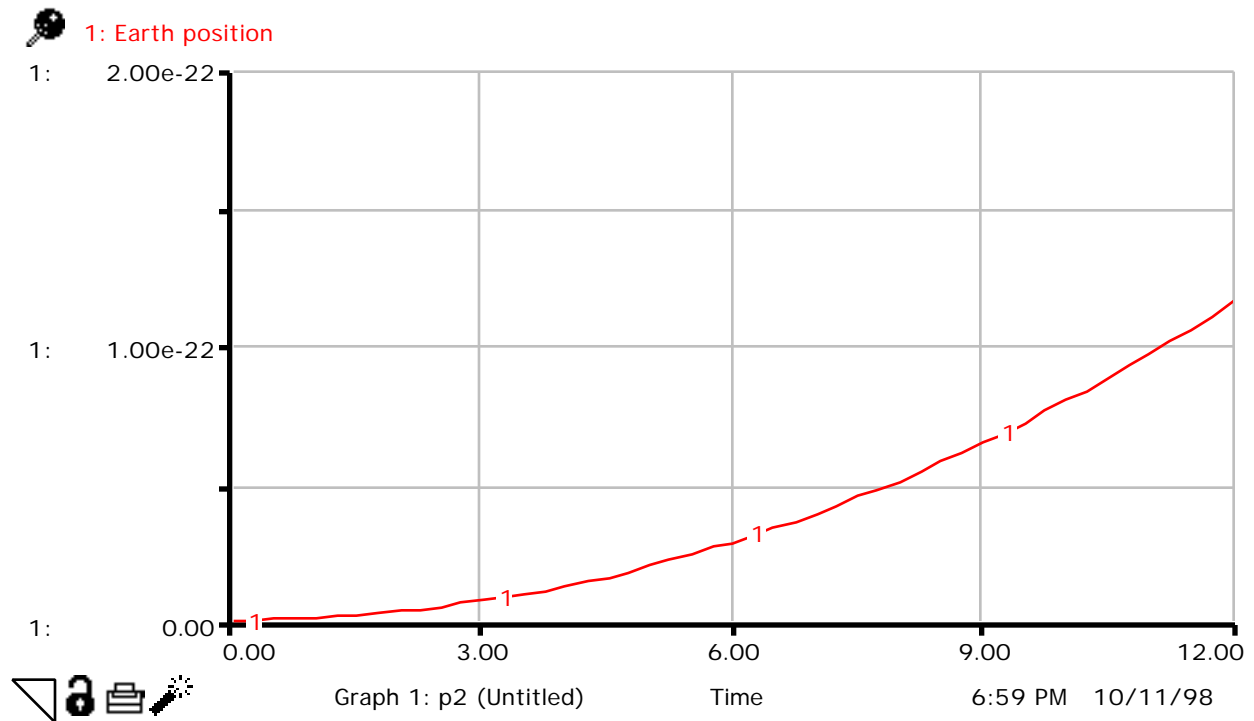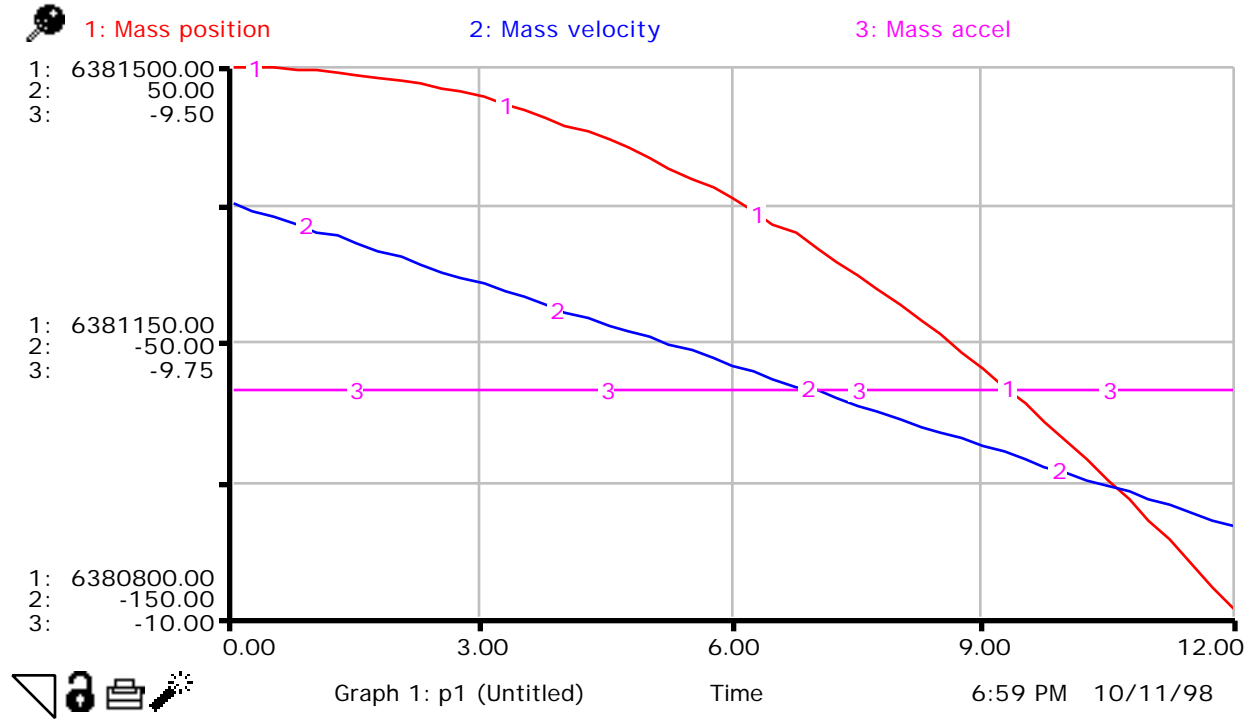
```
initial_pos_Earth = 0
initial_pos_mass = 6.38E6 + 1500 {m}
mass = 1
Mass_accel = -net_force/mass
net_force = 6.67E-11*(Earth_mass*mass)/(distance_between^2)
```

1: Mass position   2: Mass velocity   3: Mass accel

1:	6381500.00
2:	50.00
3:	-9.50

1:	6381150.00
2:	-50.00
3:	-9.75

1:	6380800.00
2:	-150.00
3:	-10.00

0.00   3.00   6.00   9.00   12.00

Graph 1: p1 (Untitled)   Time   6:59 PM   10/11/98

1: Earth position

1:	2.00e-22
1:	1.00e-22
1:	0.00

0.00   3.00   6.00   9.00   12.00

Graph 1: p2 (Untitled)   Time   6:59 PM   10/11/98

A-5

Appendix E

Extension - Building a Model for Coulomb's Law

Notes to the Teacher:

This extension will require that the students be aware that acceleration is dependent upon force divided by mass. When the particles have charge, the main force is an electrostatic force that follows Coulomb's Law:

$$F_{el} = \frac{k\ q_1 q_2}{r^2} \quad k = 9.0 \times 10^9\ Nm^2/C^2$$

You need this to modify the net force equation in the equation section. Both mass and static charge for each particle must appear in the model. The problem of attraction or repulsion may be avoided in a first cut by arbitrarily making the force attractive so that it parallels the gravitational force.

Annotated Student Guide:

1.   Open the Universal Gravitation Model in Stella.

2.   Pull down the **File** menu and **save as** Coulomb's Law.

3.   Add charge 1 and charge 2 converters to the model. **(See Appendix D)**

4.   Go to the map level and click on the force converter. Change the force converter equation to
          9.0 E9 * charge1*charge2/(Distance1 - Distance2)^2

5.   The mass of the bodies may have to be modified to get the mass moving in the time frame given.

6.   You may pick your own combination of mass and charge for each particle but think about the ratio that exists for an electron or a proton. Change things until you get the model working. Then vary the mass and charge ratios to find out the effect

7.   There is a problem with this model that you may want to avoid at first. Like charges repel and unlike charges attract. To keep the run similar to the Universal Gravitation Model, you may want to arbitrarily pick an attraction at first and make the model more variable later. **(See below)**

> **QEx.1:** How do mass and charge relate to acceleration?

> **The mass of each particle is inversely proportional to acceleration. The charge on each particle is directly proportional to acceleration.**

QEx.2: How does a **k** value that is larger than **G** affect the selection charge and mass values to enable a run?
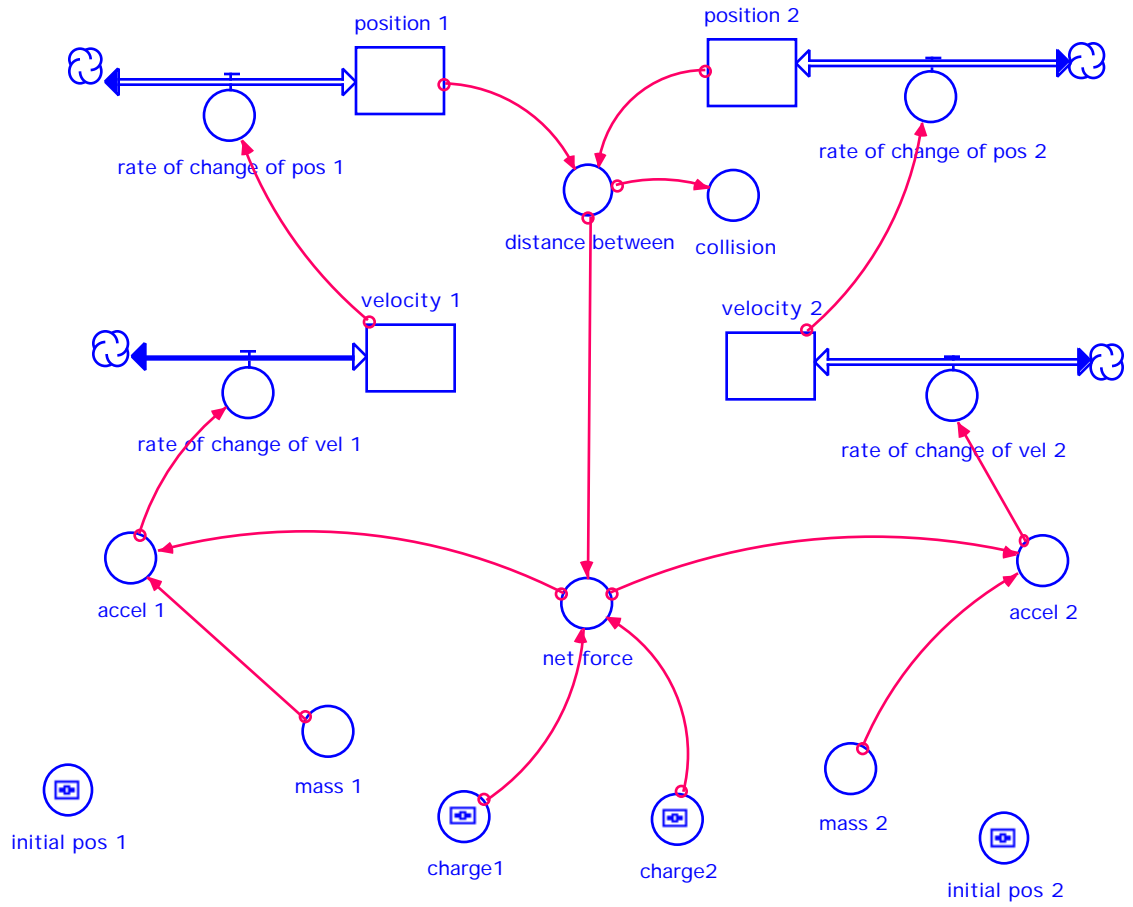
With the K value at e 9 and the G value at e-11, the new model must have a much smaller mass for the particle if the object is to be seen moving in the time frame given. Charges must be made large enough to get the mass moving.

8. Input some numbers into the model (charge, mass, position, etc.) and run it.

QEx.3: Is the output graph what you expected? Why or why not? **(See below)**

If the mass to charge ratio is selected carefully enough, you will get the particles converging in the Coulombs model in the same fashion that they moved in the Gravitational model. The cause of the force may be different and the scale may be different but the force is still a force and it will affect the mass according to F = ma.

Coulomb's Law Model

position 1

position 2

rate of change of pos 1

rate of change of pos 2

distance between

collision

velocity 1

velocity 2

rate of change of vel 1

rate of change of vel 2

accel 1

accel 2

net force

mass 1

initial pos 1

charge1

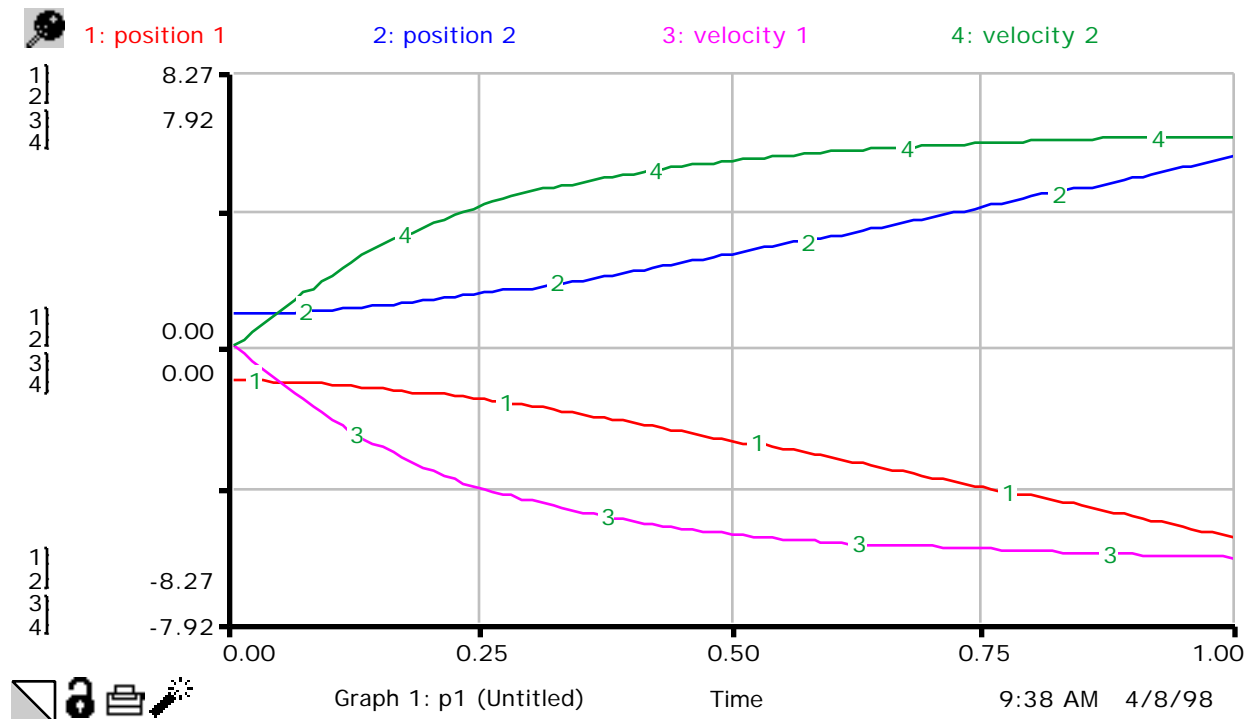charge2

mass 2

initial pos 2

Coulomb's Law Equations

```
position_1(t) = position_1(t - dt) + (rate_of_change_of_pos_1) * dt
INIT position_1 = initial_pos_1
rate_of_change_of_pos_1 = velocity_1
position_2(t) = position_2(t - dt) + (rate_of_change_of_pos_2) * dt
INIT position_2 = initial_pos_2
rate_of_change_of_pos_2 = velocity_2
velocity_1(t) = velocity_1(t - dt) + (rate_of_change_of_vel_1) * dt
INIT velocity_1 = 0
rate_of_change_of_vel_1 = accel_1
velocity_2(t) = velocity_2(t - dt) + (rate_of_change_of_vel_2) * dt
INIT velocity_2 = 0
rate_of_change_of_vel_2 = accel_2
accel_1 = -net_force/mass_1
accel_2 = net_force/mass_2
charge1 = 1e-4
charge2 = 1e-4
collision = if (distance_between<.3) then pause else 0
distance_between = ABS(position_2-position_1)
initial_pos_1 = -1
initial_pos_2 = 1
mass_1 = 1
mass_2 = 1
net_force = 9E9*charge1*charge2/distance_between^2
```

Coulomb's Law Graph

1: position 1   2: position 2   3: velocity 1   4: velocity 2

Graph 1: p1 (Untitled)   Time   9:38 AM   4/8/98

Extension - Building a Model for Coulomb's Law

1.   Open the Universal Gravitation Model in Stella.

2.   Pull down the **File** menu and **save as** Coulomb's Law.

3.   Add charge 1 and charge 2 converters to the model.

4.   Go to the map level and click on the force converter. Change the force converter equation to
$$9.0 \text{ E9} * charge1*charge2/(Distance1 - Distance2)^2$$

5.   The mass of the bodies may have to be modified to get the mass moving in the time frame given.
6.   You may pick your own combination of mass and charge for each particle but think about the ratio that exists for an electron or a proton. Change things until you get the model working. Then vary the mass and charge ratios to find out the effect

7.   There is a problem with this model that you may want to avoid at first. Like charges repel and unlike charges attract. To keep the run similar to the Universal Gravitation Model, you may want to arbitrarily pick an attraction at first and make the model more variable later.

> **Q3.1:** How do mass and charge relate to acceleration?

> **Q3.2:** How does a **k** value that is larger than **G** affect the
>   selection charge and mass values to enable a run?

8. Input some numbers into the model (charge, mass, position, etc.) and run it.

> **QEx.3:** Is the output graph what you expected? Why or why not?