CoreModels

# Radioactive Decay

A Core Learning Goals Activity for Science and Mathematics

**Summary:** Students will use a STELLA model to understand the relationship between decay constants and half-life.

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# Radioactive Decay
# Teacher Guide

## Purpose:

The purpose of this activity is to introduce the students to the concept of radioactive decay and half-life. First, the students are given M&M’s in a shoebox. They are told to shake the box and remove all M&M’s with the letters pointing up. After repeating this 12 times, they plot their data points and see the shape of an exponential decay curve. They also see how many shakes are needed to remove half of the M&M’s. This introduces them to the concept of half-life. (Note: This activity has been published elsewhere, but is not copyright protested.)

Next, they create a STELLA model to represent the same type of experiment with dice. Now they can simulate three experiments, each of which removes 1/2 or 1/3 or 1/6 of the dice. Plotting the dice remaining for each removal fraction gives the students the opportunity to determine the relationship between the removal fraction and the half-life.

In the third activity, the students transfer their learning to a real radioactive isotope, P-32. Using an isotope with a known half-life and decay constant gives the students the opportunity to verify the model. There are also two supplemental activities in which students may compare isotopes and look at nuclear series decay.

## Teacher Notes:

There are three parts to this activity. Each part has a Student Guide with directions and a Student Answer sheet for the student to write on. To conserve paper, you could photocopy a class set of Student Guides.

**Section I.** Have M&M’s and shoeboxes on hand so that each pair of students can have a box of 100 M&M’s.

**Section II.** Two versions of page S-3 of the Student Answers have been provided so that you can choose the amount of graph labeling support to offer your students.

Introduce radioactivity to the class before starting Section II so the students understand the role the dice are playing. Here is a possible script:

> In the following Stella model, we are going to use dice to represent special atoms which undergo what is called "radioactive decay." The elements which are made from these special atoms are called "radioactive" and the process of decaying is called "radioactivity." Radioactive atoms are used in nuclear bombs as well as in atomic reactors to generate electricity. What makes these atoms radioactive is an "improper" number of subatomic particles, namely protons and neutrons. All atoms are comprised of protons, neutrons, and electrons. The protons and neutrons are found in the nucleus of the atoms, with electrons traveling around in "orbitals" or "shells" outside the nucleus. Every atom of a particular element will have the same number of protons, which is what gives the element its distinct properties and makes it different from any other element. But not every atom of that element has to have the same number of neutrons. Atoms which have the same number of protons but different number of neutrons make up different isotopes of the element. The stability of any atom is due to the proton to neutron ratio. If an atom has too many or too few neutrons for the number of protons it contains, it will undergo a reaction or series of

reactions to correct the proton/neutron ratio. These nuclear reactions are called radioactivity. What actually happens to the atom is that its nucleus gives off subatomic particles, such as alpha particles (the nuclei of helium atoms) or beta particles (electrons which originate in the nucleus), so as to correct the proton/neutron ratio and become "stable". When an atoms undergoes radioactive decay, it will change into a new atom of a different element, since the number of protons contained in the nucleus changes. Over a period of time, the number of atoms which have changed will increase and the original old undecayed atoms will decrease. The time required for half of the original atoms to decay into the new ones is called the half life. This time varies for each kind of radioactive element. In the experiment using M&M's, an M&M facing up represented the decayed atoms, and the M&M's facing down represented those atoms which hadn't yet decayed. Since each time we shook the box we removed one half of the M&M's, the half life for our "radioactive" M&M atoms was one shake of the box, the time required to remove one half of the "atoms". In this dice model we will not always be removing one half of the dice each time we "shake". So one "shake" will not always be the half life of our "radioactive" dice atoms. But the meaning of the term "half-life" is still the same, namely half-life is the time it takes for one half of the "radioactive" dice to "disappear" (change into "new atoms" and therefore be removed from the box).

**Section III.** Up to this point, there has been no need for exponential decay formulas. The students simply remove a certain fraction of the objects representing a radioactive isotope. In Section III, we have the opportunity to verify that removing the appropriate fraction of P-32 does produce the known half-life of P-32.

If any students are interested in the exponential decay equation which can be used to calculate the half-life given a decay constant or to calculate the decay constant given the half-life, you may want to give them this script.

- The decay constant can be calculated by using the formula: decay constant = .693/half-life. This formula comes from the equation for exponential decay.

- $N = N_0 * e^{-kt}$ where $N_0$ is the original quantity of radioactive material, e is the natural number 2.71828..., k is the decay constant and t is time.

- Since half-life means that half of the radioactive material is remaining, we would say $N_0/2 = N_0 * e^{-kt}$.

- Simplifying, we would get $1/2 = e^{-kt}$.

- Using natural logs would yield ln(1/2) = -kt.

- Since ln(1/2) = -0.693, we would get -0.693 = -kt.

- Solving for k (the decay constant) would give us k = 0.693/half-life.

Calculus students will understand that removing P-32 at the rate of k*P-32 is the same as solving the differential equation dN/dt = kN. The solution for that problem is $N = N_0 * e^{-kt}$. Since most chemistry students have not studied calculus, you may want to surprise them by letting them know that their STELLA model is solving a calculus problem.

# Radioactive Decay
## Section I
## Teacher Answer Key and Hints

**Q1.** In this lab, the M&M's represent **a radioactive isotope**.

**Q2.** When an M&M is facing upward, it is removed from the box because it has **decayed** and turned into a new kind of atom.

**Q3.** The time it takes to remove exactly 1/2 of the M&M's is called the **half-life**.

**Data:**

| Time (trial) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number M&M's in box | 100 | | | | | | | | | | | | |

**Graph:**

**Q4.** Using your graph, determine how many time trials it took to remove 50 M&M's.

**Answers will vary.**

**Q5.** Suppose that an M&M was shaped like a cube and only one side was marked with M&M. Would it take more, less or the same amount of time trials to remove half of the M&M's? Explain your reasoning.

**It would take more time because you are removing fewer M&M's each trial.**

# Radioactive Decay
# Section II
# Teacher Answer Key and Hints

**Q1.** Approximately what fraction of the dice will be removed each time in Experiment 1? **1/2**

**Q2.** Approximately what fraction of the dice will be removed each time in Experiment 2?
**1/3 or 2/6**

**Q3.** Approximately what fraction of the dice will be removed each time in Experiment 3? **1/6**

**Q4.** How would you use Removal Fraction and Dice Remaining to calculate Number Removed?
**Number Removed = Removal Fraction * Dice Remaining**

**************************TEACHER NOTES Q5**************************

Students may observe that the expected half-life for Experiment 1 is 1 trial, but the graph hits 500 dice remaining at approximately 1.5 trials. The real explanation for this is that the model is using a dt=0.25 in order to get a smooth curve, but as a result the model is not an exact representation of the shake and removal procedure. To model the procedure more closely, you can have the students go to the Time Specs option under the Run menu and change dt to 1. When they run the model, they will see a "connect the dot" graph in which a line segment starts at a y-intercept of 1000 and ends at the point (1 trial, 500 dice remaining) giving a half-life of exactly 1 trial. In order to determine whether to bring this up with your class, consider their ability to deal with abstract concepts.

**************************TEACHER NOTES Q5**************************

**Q5.** The M&M activity had a removal fraction of 1/2. Removing all dice with an even number of spots also gives a removal fraction of 1/2. Compare the graph you drew for the M&M activity with the graph on your computer screen.

a) Are the scales the same on the two graphs? Consider both axes in your answer and explain the similarities and differences in the context of the activities.

**The scales on the y-axis are not the same because the M&M activity started with only 100 pieces whereas the Dice Model started with 1000 pieces. The scales on the x-axis are the same because each experiment ran for 12 trials.**

b) Are the shapes of the curves the same? Explain why they should or should not be the same.

**The shapes of the curves are the same because we were using the same removal fraction (1/2) in both activities.**

c) Which part of the curve shows the fastest change?

**The steepest portion of the curve shows the fastest change.**

d) How is the steepest part of the curve related to the number of dice being removed from the box in a single "shake and removal" trial?

**When the curve is its steepest, the greatest number of dice are being removed in a single "shake and removal" trial.**

e) Reading from the graph for the M&M experiment, what is the half-life measured in "shake and removal" trials?

**Student answers may vary. Ideally, the answer will be close to 1 trial, but it may be less than or more than 1 trial depending on how the shake turned out.**

f) Reading from the graph for Experiment 1 of the Dice Model, what is the half-life measured in "shake and removal" trials?

**The half-life is about 1.5 "shake and removal" trials.**

**************************TEACHER NOTES Q6**************************

If students are unsure what to write for Q6, suggest the following questions.

a) Is the general shape of each curve the same?
b) What kind of function has this shape? (linear, exponential, power, ...)
c) How is the steepness of the curve as it leaves the y-axis related to the removal fraction?
d) How is the removal fraction related to the slope of the curve as it leaves the y-axis?

**************************TEACHER NOTES Q6**************************

**Q6.** Describe in full each curve on the graph.

**Each curve represents the number of dice remaining in the box after a certain number of trials. The slope of each curve represents the number of dice removed per trial. The graphs show that the number of dice remaining is decreasing at an ever decreasing rate for each experiment. Each curve represents an exponential function.**

**The curves demonstrate that the higher the removal fraction, the steeper the curve will be showing that a greater number of dice are removed in a single trial. The smaller removal fractions result in less steep curves since fewer dice are removed in a single trial.**

**Q7.** Which of the removal fractions causes the dice to disappear at the fastest rate? **1/2**

**Q8.** Which of the removal fractions results in the shortest half-life? **1/2**

**Q9.** Which of the removal fractions causes the dice to disappear at the slowest rate? **1/6**

**Q10.** Which of the removal fractions results in the longest half-life? **1/6**

************************TEACHER NOTES Q11************************

It may be necessary to remind the students to re-read the introduction to find the units for measuring half-life.

************************TEACHER NOTES Q11************************

**Q11.** From the graph, estimate the half-life (including the units) and fill out the table below.

| Removal Fraction | Half-Life |
|---|---|
| 1/2 | **a little more than 1 shake and removal trial (1.4)** |
| 1/3 | **just slightly over 2 shake and removal trials (2.1)** |
| 1/6 | **a little more than 4 shake and removal trials (4.2)** |

************************TEACHER NOTES Q12************************

It may be necessary to review the concept of inversely proportional with your students. y is inversely proportional to x if y * x = constant.

************************TEACHER NOTES Q12************************

**Q12.** What is the relationship between half-life and the removal fraction?

> **The longer the half-life, the smaller the removal fraction. Half-life and the removal fraction are inversely proportional to each other.**

**Q13.** Suppose we wanted to remove 1/8 of the dice on each shake. Where on the comparative graph would the curve appear relative to the three curves we already have showing?

> a) Sketch the comparative graph on the axes provided, labeling each curve with its corresponding removal fraction.
> b) Sketch your prediction and label this curve 1/8.

**See Appendix 2**

**Q14.** Using the graph from **Q13**, sketch the curve that might appear if you used a removal fraction of 1/4. Label this curve 1/4.

**See Appendix 2**

Use the model to check your predictions in **Q13** and **Q14**.

**Q15.** Give a written description of a rule regarding the size of the removal fraction and the shape of the dice remaining curve.

> **The larger the removal fraction, the steeper the curve will be at the beginning because a larger fraction of the dice are removed, making the dice remaining number drop quickly.**

# Radioactive Decay
# Section III
# Teacher Answer Key and Hints

**Q1.** Write the nuclear equation for the P-32 reaction and label the parent nuclide and the daughter nuclide.

$$^{32}_{15}\mathrm{P} \longrightarrow {}^{31}_{15}\mathrm{P} + {}^{1}_{1}\mathrm{H} + {}^{0}_{-1}\mathrm{e}$$ **(the intermediate step)**

$$^{32}_{15}\mathrm{P} \longrightarrow {}^{32}_{16}\mathrm{S} + {}^{0}_{-1}\mathrm{e}$$ **(the parent nuclide and daughter nuclide)**

**parent** **daughter**

**Q2.** What should the Dice Remaining stock represent? **Parent Nuclide**

**Q3.** What should the Removal Fraction converter represent? **Decay Constant**

**Q4.** What should the Number Removed flow represent? **Nuclide Decay**

**Q5.** Can you estimate the half-life of P-32 from this graph? Justify your answer.

> **The half-life is some number greater than 12 days because at 12 days, the number remaining is still more than 500 grams. See Appendix 3.**

**Q6.** Referring back to your copy of the Graph Interpretation Guidelines, describe in full the decay curve for P-32.

> **P-32 decreases at an ever decreasing rate, which means that it decays fastest at the start; but as time goes on, the amount decaying decreases since there is less available to decay. See Appendix 4.**

**Q7.** How does the P-32 decay curve compare to the curves from the dice model?

> **The shapes seem similar until we realize that the x-axis scales are different. If all of the curves were on the same scale, the P-32 curve would decrease the most slowly and have the longest half-life because its decay constant is the smallest. The curves are similar over an extended time period. Each one shows rapid decline followed by slower and slower decline.**

**Q8.** From the graph, estimate the half-life of P-32 (including units).

> **The half-life of P-32 appears to be less than 15 days.**

**Q9**. Using the table, what is the half-life (including units) of P-32?

> **The half-life of P-32 is between 14.2 and 14.3 days.**

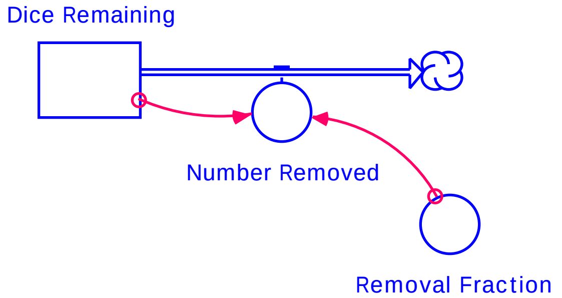
**Q10**. How do the actual half-life, the half-life estimated from the graph, and the half-life from the table compare?

**They are very close to one another.**

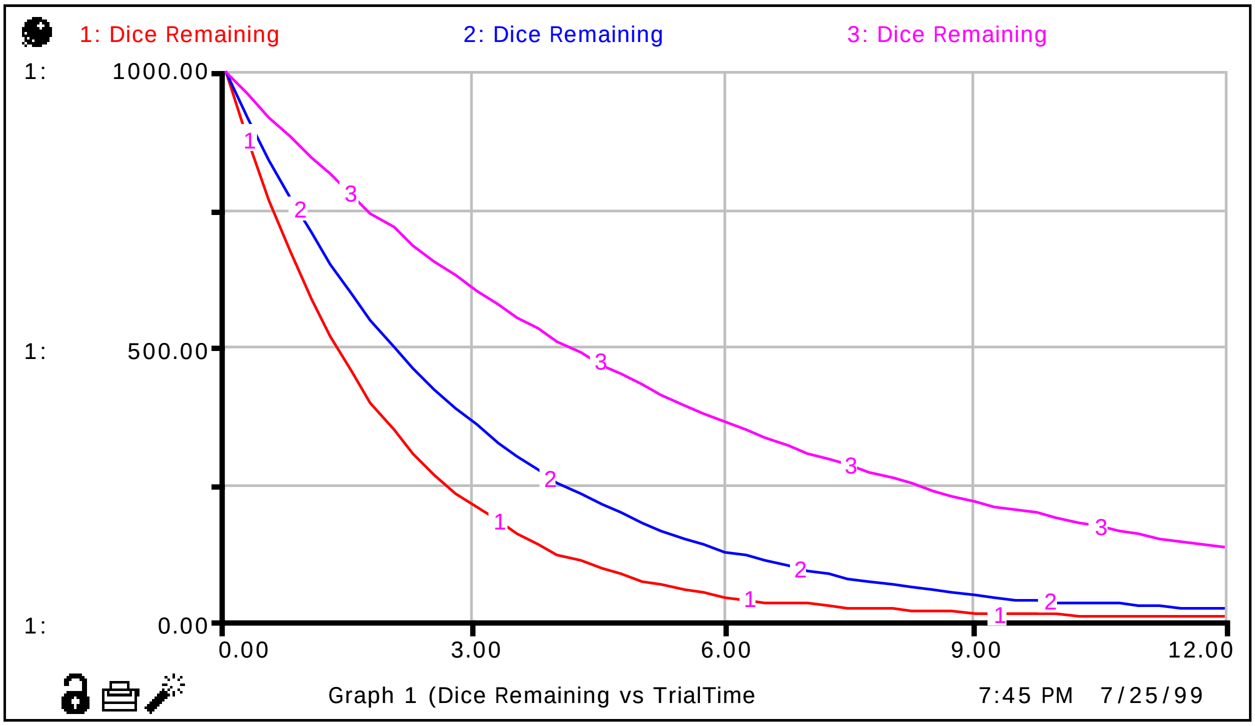
**Q11**. Does the dice model correctly demonstrate what really occurs in radioactive decay? How do you know?

**It appears to correctly demonstrate what really occurs in radioactive decay. The fact that it duplicated the half-life for P-32 is a good indicator.**

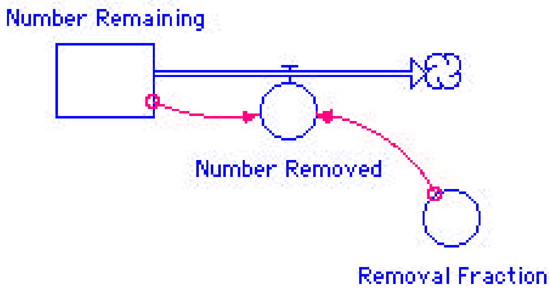
**Radioactive Decay**
**Appendix 1**



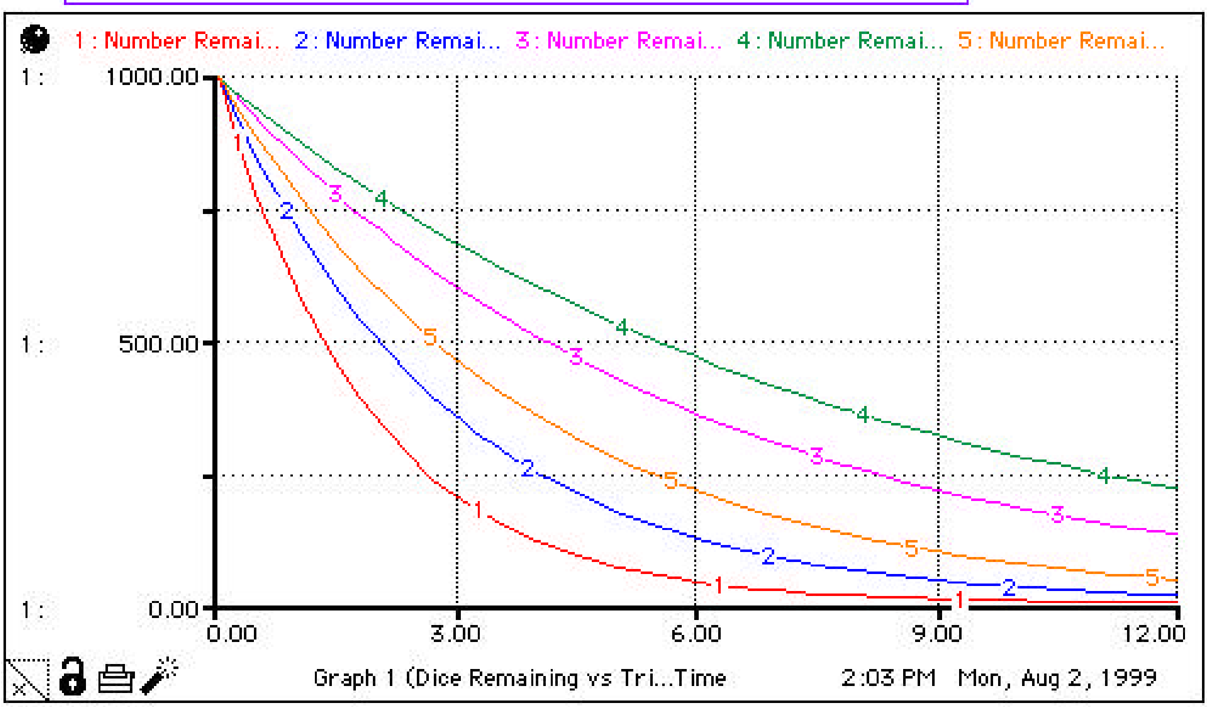

Removal Fraction 1 = 1/2
Removal Fraction 2 = 1/3
Removal Fraction 3 = 1/6



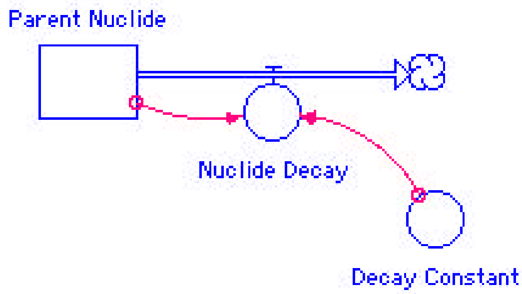
**Radioactive Decay**
**Appendix 2**



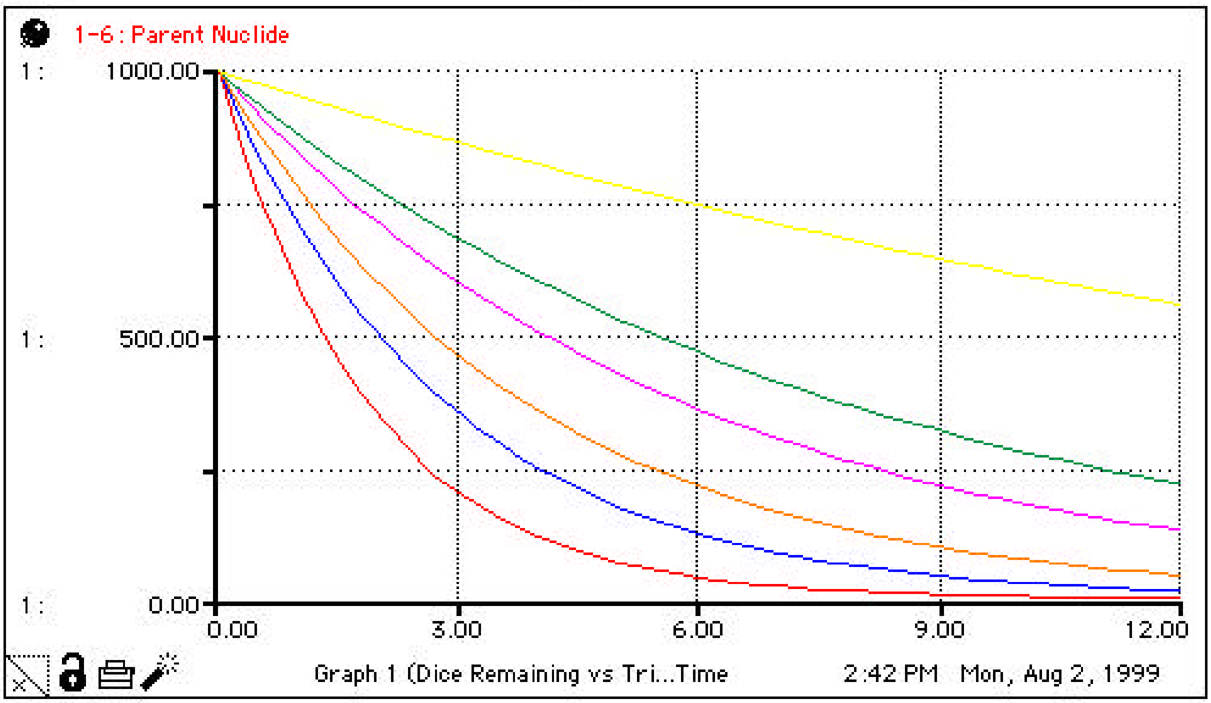

Removal Fraction 1 = 1/2
Removal Fraction 2 = 1/3
Removal Fraction 3 = 1/6
Removal Fraction 4 = 1/8
Removal Fraction 5 = 1/4



# Radioactive Decay
## Appendix 3




Removal Fraction 1 = 1/2
Removal Fraction 2 = 1/3
Removal Fraction 3 = 1/6
Removal Fraction 4 = 1/8
Removal Fraction 5 = 1/4
Decay Constant = 0.0485



# Radioactive Decay
# Appendix 4

## P-32 Nuclear Decay